CVJMT 2023

Middle School Problems and Solutions

These are the solutions to the elementary school problems for CVJMT [1] 2023. These solutions include the following in this order : Mental Math, Paper Math, Geometry & Number Theory, and finally, Algebra & Probability.

Mental Math was included in CVJMT [1] because it is an essential skill for everyday life and enables quick and accurate calculations, making it a valuable tool for solving common problems in a time-efficient manner. During the tournament, 20 mental math problems were issued with a time limit of 15 minutes to solve.

Paper Math is included in CVJMT because, like mental math, paper math is an essential part of every-day life. In many real world situations, mathematical problems can be complex and require multiple steps to solve. Paper math allows for a more organized and step-by-step approach, where intermediate results can be recorded and used in subsequent calculations. Additionally, in many fields, such as science and engineering, paper math is used to communicate complex mathematical concepts and results. Equations and calculations are written down on paper and used to explain and justify scientific discoveries and technological advancements. In CVJMT, we issued 20 paper math problems with 30 minutes to complete.

Geometry and Number Theory were categories in CVJMT because these two branches of mathematics are both interesting and vital to higher-level math courses. Despite focusing on different types of mathematical objects, geometry and number theory share a fundamental approach to mathematical reasoning and the study of mathematical structures. Both fields require abstract thinking, logical reasoning, and an understanding of mathematical structures, as well as their practical applications. In the tournament, 20 Geometry and Number Theory questions were offered with a time limit of 15 minutes.

Algebra and Probability were categories in CVJMT because these two branches of mathematics are both indispensable to find unknown quantities. Algebra and Probability both require extensive skills in manipulating equations and formulas. It tests your skills of analytical thinking and problem solving. Ultimately, Algebra and probability are both used in many other fields of mathematics and science. For example, algebra is used in calculus, physics, and engineering, while probability is used in statistics, economics, and computer science. In the tournament, 20 Algebra and Probability questions were prompted with a time limit of 15 minutes.

[1] CVJMT - Castro Valley Junior Math Tournament

Problem 1: What is 1.23 + 45.678?

An easy way to do this in your head is to add the places together (tens place adds with other tens place). You should start with the smallest place. 45.678 has the smallest place, the thousandths place. 1.23 does not have a thousandths place. $8 + 0 = 8$. Move on to the next places. You end with 46.908.

Problem 2: How many seconds does it take to cross a 100 meter bridge if you walk at 50 meters per second?

You can use the distance-speed formula : $distance = speed \times time$
Plugging it into the equation : $100 = 50 \times time$, $time$ = 2 seconds

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Problem 3: What is x if $2^x = 64$?

2 to the power of a number is 64. A power means that you are multiplying by itself x times. If you multiply 2 by itself 6 times, then you result with 64. Therefore, $x = 6$.

Problem 4: What is 5% of 200?

5% is equivalent to 0.05. Multiplying 0.05 and 200 together results in 10

Problem 5: If one egg has enough yolk for two cakes, then how many eggs must be cracked for 5 cakes?

Conversion factor : 1 egg = 2 $cakes$
5 $cakes \times \frac{1\ egg}{2\ cakes}$ = 2.5 eggs. Since you cannot have half of an egg, the answer is 3 $eggs$.

Problem 6: How much money is 7 hundred-dollar bills, 6 ten-dollar bills, 5 one-dollar bills, 4 quarters, 3 dimes, 2 nickels, and 1 penny?

Hundred dollar bills represent $100. Ten dollar bills represent $10. One dollar bills represent $1. Quarters represent $0.25. Dimes represent $0.10. Nickels represent $0.05. Pennies represent $0.01.

Then, sum the numbers all up. $700 + 60 + 5 + 1 + 0.30 + 0.10 + 0.01$ = $766.41

Problem 7: What is 23 + 34 + 42?

Simply add the tens place and then add the ones place.
The tens place would be : $2 + 3 + 4 = 9$.
The ones place would be $3 + 4 + 2 = 9$.
The combined number would be 99.

Problem 8: What is the sum of the first ten natural numbers starting at 1?
You can use a math trick for this problem. Simply add the top and bottom numbers together. For example, 1 + 10 = 11, 2 + 9 = 11, and so on. You can see that they all equal 11. These can be called 'summation pairs'. We can see that there is 5 pairs in this series.
$5 \times 11 = \underline{55}$

Adding them manually would also get you 55.

Problem 9: What is a + b if a = b and b = -4?

If a = b, then a would also equal -4. Therefore, the answer would be $(-4) + (-4) = \underline{-8}$

Problem 10: If $\frac{1000}{100+x} = 1$, what is the value of x?

For the fraction to equal to 1, the numerator (top portion) must be equal to the denominator (bottom portion). So, $1000 = 100 + x$.

Solving for x, we find that $\underline{x = 900}$.

Problem 11: Simplify $(-2x) \times 7 - 10x$.

By the rule of *PEMDAS*, we multiply before subtraction. $-2x \times 7 = -14x$.
Next, $-14x - 10x = \underline{-24x}$

Problem 12: What is the smallest palindrome greater than 1392? A palindrome is a number that reads the same forwards or backwards, such as 3443.

Since there are four digits in 1392, that means that the palindrome really depends on the first two digits. If we are trying to go for the smallest palindrome, we could try 1331, but that number is not bigger than 1392. We can add one to the set of two digits. We end up with 1441. That is bigger than 1392. 1441.

Problem 13: How many numbers in the set below are divisible by 5?
$\{32, 10, 15, 18, 52, 30\}$

Numbers that are divisible by 5 are numbers that end with 0 or 5. 10, 15, and 30 fit that criterion. Therefore, there are 3 numbers divisible by 5.

Problem 14: How many numbers in the set below are divisible by 3?
$\{11, 33, 128, 1566, 378, 9\}$

There is no complete trick for numbers that are divisible by 3. We can look at each number individually. We know that 11 is not divisible by 3. 9 is divisible by 3. So is 33. You can tell that 128 is not. 1566 is because 15 is divisible and 66 is also divisible. 378 is also divisible because 3 is divisible and 78 is too.
The trick is taking two numbers at the same time and dividing by three. For example, for 1566, take 66 and take 15. If they are both divisible by 3, then 1566 is divisible by 3. Therefore, there are 4 numbers in this set that are divisible by 3.

Problem 15: Which two numbers have a sum of 14 and a product of 45?

By intuition, you can tell that the answer is 9 and 5.

However, you can solve with algebra. Set the two nunmbers to x and y.

$$x + y = 14 \quad \text{(Sum)}$$
$$x \times y = 45 \quad \text{(Product)}$$

$$y = 14 - x \quad \text{(from Sum)}$$
$$x \times (14 - x) = 45 \quad \text{(Plug into equation)}$$
$$14x - x^2 = 45$$
$$x^2 - 14x + 45 = 0$$
$$(x - 9)(x - 5) = 0 \quad \text{(Or use quadratic equation)}$$
$$\underline{x = 9, 5}$$

Problem 16: If $x^x = 4$, what is the value of x?

x = 2 is the only result that works.

Problem 17: Kyle reads 24 pages a day. *The Great Gatsby* has 264 pages. How many days does it take for him to finish it?

You can tell by intuition that if you divide the total number of pages by the amount of pages Kyle reads per day, you can get the number of days. 11 days.

Conversion factor : 24 pages = 1 day. 264 pages $\times \frac{1 \text{ day}}{24 \text{ pages}}$ = 11 days.

Problem 18: What is the largest area that can be enclosed within 24 feet of fencing?

You may think that the largest area is a square, but it is a circle. Since the circumference (fencing) is 24 feet, and the circumference of a circle is $2\pi r$, r = $12/\pi$.

Now, the area of a circle is πr^2. Plugging r into the area equation, we result in $\frac{144}{\pi}$.

Problem 19: Evaluate $55^2 - 54^2$.

You can do this by hand in your head (since you can't use paper for this section) if you're really good at doing arithmetic in your head, but there's an easier way to do this.

The formula is $a^2 - b^2 = (a - b)(a + b)$.

In this case, a = 55 and b = 54. Plugging that into the equation, $55^2 - 54^2$ = (55-54)(55+54) = (1)(109) = 109.

Problem 20: What is $\sqrt{2\frac{1}{4}} + \sqrt{1\frac{7}{9}}$? Express your answer as a mixed number.

You can change $\sqrt{2\frac{1}{4}}$ to $\sqrt{\frac{9}{4}}$, which is simply $\frac{3}{2}$.

You can change $\sqrt{1\frac{7}{9}}$ to $\sqrt{\frac{16}{9}}$, which is simply $\frac{4}{3}$.

You end up with 3/2 + 4/3 = 9/6 + 8/6 (common denominator) = 17/6. Then turn it to a mixed number. $2\frac{5}{6}$.

Problem 1: Evaluate $5 \times 4 + 15 - 10$.

By the *PEMDAS*[1] Rule, you multiply first, then add and subtract next. Then, simplify.

$$\begin{aligned} 5 \times 4 + 15 - 10 &= ? \\ 20 + 15 - 10 &= ? \text{ (multiply)} \\ 35 - 10 &= ? \text{ (add)} \\ 25 &= ? \text{ (subtract)} \end{aligned}$$

Therefore, the answer is 25

Problem 2: While shopping for party supplies, Bessie the Cow sees a group of cows and chickens that she can invite to her Pi Day Party. She counts 23 heads and 48 feet. How many chickens are there?

We can infer that the cows have 1 head and 4 legs. Also, chicken have 1 head and 2 legs. Using this information and the information above, we can set up two equations. Let the number of cows equal to A and the number of chicken equal to B.

$$\begin{aligned} A + B &= 23 && (1) \\ 4A + 2B &= 48 && (2) \\ \\ 2A + 2B &= 46 && 2 \times (1) = (3) \\ \\ 2A &= 2 && (2) - (3) \\ A &= 1 \\ \\ 1 + B &= 23 && \text{(Plug into (1))} \\ B &= 22 \end{aligned}$$

Therefore, there are 1 cow and 22 chicken.

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[1]PEMDAS - A guideline of what steps to do first. The list, from first to last : Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction.

Problem 3: Becky the Cow wants to invite 2 times as many cows as chickens, 3 times as many chickens as sheep, and 5 times as many sheep as pigs. Becky invites 17 pigs to the party. How many cows should she invite?

Use dimensional analysis :

Conversion factors :

- 2 cows = 1 chicken
- 3 chicken = 1 sheep
- 5 sheep = 1 pig

17 pigs $\times \frac{5\ sheep}{1\ pig} \times \frac{3\ chicken}{1\ sheep} \times \frac{2\ cows}{1\ chicken}$ = 17 $\times$ 5 $\times$ 3 $\times$ 2 cows = 85 $\times$ 6 cows = 510 cows

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Problem 4: Bella the Cow wants to build tables for the party. She doesn't have a ruler, but she knows that the length of 4 pigs standing in a line equals 7 meters. If she wants to make a table that is 20 pigs long and 12 pigs wide, what is the area of the table in square meters?

Dimensional Analysis is the way to go for this problem.

Conversion factors are listed :

- 4 pigs = 7 meters

Area is equal (20 $\times$ 12) pigs^2 = 240 pigs^2.

240 pigs^2 $\times \frac{7\ meters}{4\ pigs} \times \frac{7\ meters}{4\ pigs}$ = 735 meters^2.

Problem 5: Abby the Cow brought a group of pigs to play pie roulette. There are 35 pigs total: 15 have brown spots, 4 have black spots, and the rest have no spots. Assuming that every pig has an equal chance of being pied, what is the probability that the pied pig has no spots?

Given :

- 15 pigs with brown spots
- 4 pigs with black spots
- 35 - 15 - 4 = 16 pigs with no spots

Probability is equal to amount of pigs that are waiting to be chosen over the total amount of pigs.

$\frac{16\ pigs\ with\ no\ spots}{35\ total\ pigs} = \frac{16}{35}$.

<u>Problem 6</u>: Bobby the Cow is an extremely picky eater. He requests that his pie slice has an angle of 20 degrees. If the pie's diameter is 12 inches long, what is the area of the pie slice that Bobby gets? Leave your answer in terms of π.

Since the diameter is 12 inches long, the radius of the pie is 6 inches long. The area of a pie (circle) is πr^2.

Plugging in r = 6 into the area equation, $\pi(6)^2 = 36\pi$.

Since Bobby only wanted 20 degrees out of the whole 360 degrees of the pie, Bobby only gets 20/360 of the pie, which is 1/18 of the pie. Multiply 1/18 to the total area.

$1/18 \times (36\pi) = \underline{2\pi}$.

<u>Problem 7</u>: Alice the Cow has many hobbies. One of them is making soccer balls. She has 490 pentagonal panels and 921 hexagonal panels. Each ball needs 12 pentagonal panels and 20 hexagonal panels. How many soccer balls can she make?

Since each ball requires 12 pentagonal panels and 20 hexagonal panels, you must have those 'ingredients' to make a ball, no less, otherwise you cannot assemble a ball. A good way to see it is to see how many balls you can make using just pentagonal panels or just hexagonal panels.

490 pentagonal panels $\times \frac{\text{1 ball}}{\text{12 pentagonal panels}} = 40.833$ balls

921 hexagonal panels $\times \frac{\text{1 ball}}{\text{20 hexagonal panels}} = 46.05$ balls

* During the test, you just need the approximate values to see which is the limiting factor.

We can see from the data above that the pentagonal panels are the limiting factor. You can only make <u>40 total balls</u>.

<u>Problem 8</u>: Alice the Cow also likes selling apples. If an apple sells for \$2.50, and an Apple computer costs \$1000, how many apples will Alice need to sell to buy an Apple computer?

Conversion factors are listed:

- 1 apple = \$2.50
- 1 apple computer = \$1000.00

Dimensional Analysis is the way to go!
1 apple computer $\times \frac{\$1000}{\text{1 apple computer}} \times \frac{\text{1 apple}}{\$2.50} =$ <u>400 apples</u>.

<u>Problem 9</u>: Farmer Mark recently bought a rectangular plot of land. It has an area of 60 square feet, and it is 15 feet wide. If he wants to build a fence around the perimeter, how many feet of fencing does he need?

Using the area and one of the sides, we can solve for the other side.

Area = length $\times$ width
60 = 15 $\times$ length
length = 4 feet
Now that we know the length is 4 feet and the width is 15 feet, we can solve for the perimeter. The formula for perimeter is : (Perimeter = 2 $\times$ length + 2 $\times$ width)

$2 \times 15 + 2 \times 4 = 30 + 8 =$ <u>38 ft^2</u>.

<u>Problem 10</u>: Farmer Alex recently registered for his first bank account, and he needs to pick a PIN. It has to be a letter (A - Z) followed by 3 digits (0 - 9). How many different PINs might he choose?

The combinations of PINs are the combinations of a digit multiplied by the combinations of all the other digits. So, that means that the combination of the first digit times the second times the third times the fourth is the total combinations that the PIN can have.

The first combination consists of all the letters in the alphabet, which is 26. The second combination is the digits (0-9). There are 10 digits. The third and fourth combinations follow that.

Total combinations = $26 \times 10 \times 10 \times 10$ = <u>26000 combinations</u>.

<u>Problem 11</u>: Farmer Isabella plans to steal Farmer Alex's chickens. Isabella knows that Alex has 5 chicken coops and 45 chickens. Every chicken coop has at least one chicken, but Isabella doesn't know exactly how many chickens there are in each chicken coop. She only has time to visit two chicken coops. What is the maximum number of chickens that could be in two coops?

If there are at least 1 chicken in each coop for the 5 coops, we can maximize choosing two coops by having only 1 chicken in the other coops and having most of the chicken in one of the coops.

So, there are 1 chicken in each of 4 coops. There are then 41 chicken in the remaining coop. Choosing two coops would get you <u>42 chicken</u>.

Problem 12: The four-digit number 5X76 is divisible by 3, and the digit X is greater than 6. What is the value of X?

If the number is greater than 6, we know that the digit is either 7, 8, or 9. Because 76 is not divisible by 3, '5X' cannot be divisible by 3. That wipes 7 off the board.

So we're left with 8 and 9. Do guess and check, and you see that 9 is your answer.

Problem 13: If 1760 yards are in a mile, how many miles are in a yard?

Use dimensional analysis for this problem.

Conversion Factors :

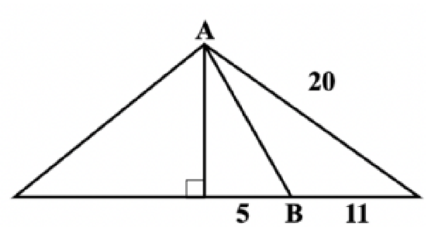
- 1760 yards = 1 mile

1 yard $\times \frac{1\ mile}{1760\ yards}$ = 1/1760 miles

Problem 14: Soolynn and Joyce are in a heated debate about their favorite numbers. Soolynn's favorite number is $\pi \approx 3.14159$. Joyce's favorite number is $\tau \approx 6.28318$. Soolynn claims that the product of the first six digits of her favorite number is greater than the product of the first six digits of Joyce's favorite number. Find the positive difference between those two values.

The problem only asks for the difference between the two numbers. τ is just 2π. So, the difference is just π.

Problem 15: Erin needs to get from point A to point B. She knows the lengths of the other sides in this diagram. What is AB?



To solve for AB, you first need to solve for AC, where C is the point where there is a right angle. For that triangle the hypotenuse is 20 and one of the leg is 16. Solve for the other leg, AC using the pythagorean theorem.

$$20^2 = 16^2 + AC^2$$
$$400 = 256 + AC^2$$
$$AC^2 = 144$$
$$AC = 12$$

Now that we know AC is 12. We can look at the triangle, ABC. Do the pythagorean theorem again where the hypotenuse is AB and the two legs are 12 and 5.
You get AB = 13.

Problem 16: Katie is super enthusiastic about the Fibonacci sequence. It begins 1, 1, 2, 3, 5, 8, . . . What is the 12th term of the Fibonacci sequence?

The Fibonacci's sequence is taking the previous two terms and adding it to get your new term. You can continuously do this from the 6th term, which is 8. The 7th term is 5 + 8 = 13. The 8th term is 21. The 9th term is 34. The 10th term is 55. The 11th term is 89. The 12th term is 55 + 89 = 144.

Problem 17: If $2^x \times 3^y \times 7^z = 252$, what is the value of $2x + 3y + 7z$?

You can solve for the variables by solving for the roots of the total number, 252. The roots of 252 are : 2, 2, 3, 3, and 7. There are two 2s, two 3s, and one 7.

That means that x = 2, y = 2, and z = 1.

Plugging that into the equation, 2(2) + 3(2) + 7(1) = 4 + 6 + 7 = 17

Problem 18: Erin is on an adventure! She starts at M in the diagram and can either move to an adjacent letter to the right, left, up, or down. How many paths can she make that spell the word MATH?

Split the diagram into a quarter of the diagram.

There is 2^3 permutations just for this quarter. Since there is one path that overlap for each quarter, you subtract 4 from the total permutations. You have 32 - 4 = 28 permutations (paths).

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H
T H
A T H
M A T H
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      H
    H T H
  H T A T H
H T A M A T H
  H T A T H
    H T H
      H
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Problem 19: Given that $3^x = x^3$, what is one possible value of x?

If you see that x is the base and the exponent at the same time, we can see that x = 3.
$3^3 = 3^3$

Problem 20: The two roots of the quadratic equation $ax^2 + bx + c = 0$ are given by the equations

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

What is the product of the two roots?

You can manually multiply them together, but a trick to finding the product of the two roots in a quadratic equation is simply c/a.

Problem 1: Let's play a game!

1. Pick a number between 1 and 25
2. Add 9 to it
3. Multiply the result by 3
4. Subtract 6
5. Divide by 3
6. Subtract your original number

What number do you have?

Let's say the number that you chose was x. Add 9 to it. You get 'x + 9'.

Then, multiply the result by 3. You get : 3(x + 9) = 3x + 27

Subtract 6. 3x + 27 - 6 = 3x + 21

Divide by 3. $\frac{3x+21}{3}$ = x + 7

Subtract original number (x). x + 7 - x = 7.

Problem 2: The ages of Liz, Tom, and Kara are 4, 5, and 6 respectively. In how many years will the sum of their ages be 60?

You can label the years that pass by as 'x'. Since there are three people, the addition would be 3x. The sum of their ages are 15.

Set the equation : 15 + 3x = 60.
Solve that and you get x = 15. It takes 15 years for the sum of their ages to be 60.

Problem 3: When paper strip a with a length of 23 inches is attached onto paper strip y, the new strip of paper has a length of 31 inches. If the length of the overlapping part is 2 inches, how long is paper strip y?

The equation goes : 23 + y - 2 = 31. The sum of the two lengths minus the overlapping part is the total length (31 inches)
y + 21 = 31, y = 10

Problem 4: Anna was given homework for summer break. She finished 5/7 of it after 25 days. If she continues working at the same rate, how much longer will it take for her to finish the homework?

This is a dimensional analysis problem. The conversion factor is : 5/7 of homework = 25 days. The starting quantity is 100% of homework, since we're trying to find how long it takes for Anna to complete the homework.

1 of homework $\times \frac{25 \text{ days}}{5/7 \text{ of homework}}$ = 35 days. So, it takes her 35 days to do the entire homework. Since she has already done it for 25 days, it takes her 10 more days to finish the rest of the work.

Problem 5: According to the poster on a printing shop, it costs $1 to print each page. However, they offer a 15% discount to customers who print at least 500 pages. John was planning to print less than 500 pages, but he realized that the total cost would be cheaper if he printed 500 pages. What is the minimum possible number of pages John was planning to print?

If John printed 500 pages, the discount would minimize the cost to 425 dollars. We are trying to find the minimum amount of pages that are less than 500 that costs more than the cost for 500 pages, which is 425 dollars. Any amount under 500 has no discount. So, the minimum amount of pages that John was planning to print was 426 pages.

Problem 6: Eva made a sugar solution with 20g of sugar and 100g of water. She then added 5g of sugar into the solution. How much water in grams should be added to the solution order to keep the original ratio of sugar to water?

The ratio of sugar to water is 20 : 100, which is 1 : 5. After the addition of 5g of sugar, there is 25g of sugar. To keep that ratio, there would need to be 25 $\times$ 5 = 125g of water. That is 25 more grams of water.

Problem 7: Car A and Car B are in a race. Car A is driving at 60 mph and is 30 miles away from the finish line. If Car B is 40 miles away from the finish line, what is the minimum speed that Car B must be driving at in order to not lose the race? Assume that the speed is constant throughout the race.

Using the equation, d = vt for Car A, 30 = 60t, t = 0.5 hours. Since it takes 0.5 hours for Car A to reach the finish line, Car B wants to get there at the same time or earlier. Using the equation for Car B, d = vt, 40 = v(0.5), v = 80 mph. Car B needs to travel 80 mph to finish faster than Car A.

Problem 8: A card is drawn from a standard 52-card deck. What is the probability that it is a red face card or the ace of hearts? Jacks, Queens, and Kings are considered face cards.

There are 6 red face cards, and 1 card that is an ace of hearts. That is a total of 7 cards that are within this probability. Since there are a total of 52 cards, the probability is 7/52.

Problem 9: If there are 2 green and 2 blue candies in a bag, what is the probability of getting blue candies twice in a row?

The probability that you get a blue candy the first time is 2/4 or 1/2. The probability of getting it again is 1/3, since there are only one blue candy left in the bag and that there are 3 total candies in the bag. The total probability of getting it twice is the two probabilities multiplied together, which is 1/6.

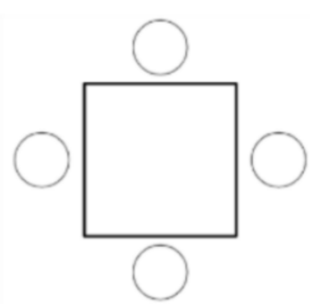
Problem 10: If the sum of two different positive integers is 41, find the maximum possible value of their product.

The maximum products happen if the two numbers are as close to each other as possible if they have a fixed sum. Those numbers would be 20 and 21. The maximum product is 420.

Problem 11: Bill has 4 red socks, 5 pink socks, 9 black socks, and 17 purple socks. What is the minimum number of socks Bill needs to pull from his sock closet to guarantee that he has at least 5 pairs of matching socks?

To get 4 pairs of matching socks, which is 8 socks, you could have two of each. To get the maximum number of socks before getting the fifth pair, you would have three of each sock, which is 12 socks. Then, choosing any random sock would get you the fifth pair. The answer is 13 socks.

Problem 12: Laura, her brother, and their parents are dining at a restaurant. The arrangement of their seats is shown in the diagram below. If they pick their seats randomly, what is the probability that Laura sits opposite of her brother?



If they pick seats randomly, you can place Laura at any spot. There is always only one spot on the left of Laura. There are 3 total open spaces. So, the probability that her brother sits next to her is 1/3.

Problem 13: The distance between Eric and the train station is 8 cm on a map with a scale of 1 : 25000. If Eric walks at 3 km/h and leaves his home at 8:30 AM, when will he arrive at the train station?

If the distance between Eric and the train station is 8cm on the map, then in real life, that distance is 25000 $\times$ 8 cm = 200,000 cm $\times$ $\frac{1\text{m}}{100\text{ cm}}$ $\frac{1\text{km}}{1000\text{ m}}$ = 2 km.

Using the equation, d = vt, 2 km = 3 km/hr (t), t = 2/3 hours = 40 minutes. If you add 40 minutes to 8:30 AM, you get 9 : 10 AM.

Problem 14: The teacher is calculating the average score of each row in his classroom. In a row where there are 5 students sitting, the first three students' average score is 83, while the last three students' average score is 74. If the student in the middle of the row has a score of 81, what is the average score of this row?

To find the average score of the row, you find the total score of the row. Then divide that total score by the number of people that are in the row, which is 5.

The first three students' average is 83, meaning the total score within the first three are 249. Since the middle student got an 81, the total score between the first two students are 168.

The last three students' average score is 74, meaning the total score is 222. Since the middle student scored an 81, the total score within the last two students are 141.

THe total score is 141 + 168 + 81 = 390. The average would be 78.

Problem 15: Jordan is taking a taxi to catch a tour bus that has left without her. The taxi driver told her, "If I drive at 80 km/h, we can catch the tour bus in 1.5 hours, but if I drive at 90 km/h, we can catch it in 40 minutes." Based on the conversation, what is the speed of the tour bus in km/h?

Let's say that the speed of the bus is 'x'. We know that the speeds that the taxi is going is faster than 'x'. We can set up the equation : d = vt. Let d be the distance between the bus and the taxi initially.

For the first instance, d = (80 - x)(1.5) = 120 - 1.5x
For the second instance, d = (90 - x)(0.66666) = 60 - 2/3 x

We can set d equal to each other : 120 - 1.5x = 60 - 2/3 x, 60 = (3/2 - 2/3)x, 60 = 5/6 x. We can solve for x and find that x = 72. The speed of the tour bus was 72 km/hr.

Problem 16: A truck driver is hired to deliver 500 glass bottles. He earns \$1.50 for each successful delivery but pays \$13.50 for each broken glass. If he earned a total of \$675 in the end, how many glass bottles were broken?

You can set up a system of equations, where x is the successful glasses and y is the broken glasses. The equation would be $1.5x - 13.5y = 675$ and $x + y = 500$. This is because for every successful delivery, you add 1.5 dollars and for every broken one, you subtract 13.5 dollars. Then, the sum of the glasses are 500.

You can solve the system of equation, and $x = 495$ and $y = 5$. 5 bottles were broken.

Problem 17: According to the directions on a bottle of liquid laundry detergent, the amount detergent used each time should be 1/3 of the cap. However, Katie mistakenly read it as 1/2. When she realized this, she had already used the detergent 12 times, which is 1/3 of the bottle. If she follows the directions correctly every time afterwards, how many more times can it be used?

Using 1/2 the cap 12 times is a total of 6 caps. If that is 1/3 of the bottle, then you can do the same 3 times. There are a total of 18 caps, which means that she can use 12 more caps. Since she should use 1/3 of the cap per usage, she can use it 36 more times.

Problem 18: The plan for a tree planting event was to plant 20 trees per hour. However, during the actual event, 8 more trees were being planted per hour. As a result, the event ended 2 hours sooner than planned. How many trees were planted during the event?

We can use the equation : $d = vt$. We can use x as the amount of trees that were to be planted. We can use the variable y as the amount of time it takes to plant.

The first instance, the normal way, goes : $d = vt$, $x = 20(y)$
The second instance, the faster way, goes : $d = vt$, $x = 28(y - 2)$

You can set $x = x$, so $20y = 28y - 56$, $8y = 56$, $y = 7$. Originally, it was supposed to take 7 hours for the event to complete, but now, with a faster rate, it takes 5 hours for the event to complete.

Plugging that into the equation, we can get $x = 140$, meaning that there were 140 trees planted during the event.

Problem 19: The photocopier outputs 100 sheets of paper per minute. If the photocopying time exceeds 30 minutes, the machine will stop for 10 minutes. Then it will continue to operate at the same speed. If Tom uses this photocopier to print all the pages at 14:30, it will stop printing at 17:00. How many sheets of paper did Tom print during this time?

From 14:30 to 15:00, the printer has a session of printing. After a 10 minute rest, the next session is 15:10 to 15:40. The third session is 15:50 to 16:20. The fourth session is 16:30 to 17:00. There are 4 sessions.
In each session there are 3000 pages printed because 30 minutes $\times$ 100 papers per minute. Since there are 4 sessions, there are 12000 pages printed.

Problem 20: Emily, Thomas, and George each spent 20 minutes, 25 minutes, and 100 minutes on their homework respectively. How much time would be needed if all three of them were doing the homework together?

We can solve for this problem using their combined rates or speeds that they do their homework with. The rates are 3 homeworks per hour, 2.4 homeworks per hour, and 0.6 homeworks per hour respectively. Their combined rates are 6 homeworks per hour. Since there is only one homework, it takes them 10 minutes to complete it.

Problem 1: Two angles in a triangle measure 12 degrees and 34 degrees. What is the measure of the third angle?

Triangles have a property that the sum of all of the internal angles are 180 degrees. Let the missing angle be X.

X + 12 + 34 = 180 , X = 180 - 46 , X = 134

Therefore, the measure of the third angle is 134 degrees.

Problem 2: What is the perimeter of a hexagon with side lengths of 4, 5, 8, 4, 5, and 8?

The perimeter is simply the sum of all of the side lengths, which are all listed above. The perimeter is 4 + 5 + 8 + 4 + 5 + 8 = 34.

Problem 3: How many lines of symmetry are in an equilateral triangle?

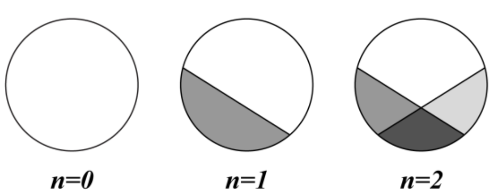
There are 3 lines of symmetry because you can split it straight down the middle for every side the triangle has. The triangle has three sides, and there is 3 lines of symmetry.

Problem 4: What is the maximum number of right angles that a pentagon can have?

A pentagon has 5 sides. In this equation : (n - 2) $\times$ 180, we can find the total internal angle of the pentagon. n is the amount of side lengths. (5-2) $\times$ 180 = 540 degrees.

The angles have enough quantity to have more than 5 right angles, but you can't have any angles left over, so if you have four right angles, the final angle has to be 180 degrees. You can't have a 180 degree internal angle because that would just be a straight line. Then, you can only have 3 right angles.

Problem 5: What is the maximum number of pizza slices that can be made with only 4 straight cuts? An example with 0, 1, and 2 cuts is shown below.



The goal is to cut the most slices with 4 cuts. The first two cuts were already shown. The strategy is to cut straight down without going through already pre-existing intersections because you want to make the most of the cuts. You end up with 11 slices.

Problem 6: What is the largest number smaller than 1000 that leaves a remainder of 11 when divided by 17?

If the remainder is 11, then the maximum number that is multiplied by 17 is 1000 - 11 = 989. If you divide 989 by 17, you get 58 with a remainder of 3. Since you cannot have a remainder in these circumstances, the answer would be 17×58 = 986.

Problem 7: What is the smallest possible product of a two-digit number and a three-digit number obtained from five distinct digits?

Since you are aiming for the smallest numbers, you want the smallest digits. Those digits are 0, 1, 2, 3, and 4. Since you can't put 0 as your first digit of a number, it is most logical to put it as the second digit of the three digit number. So you are now left with _0 $\times$ ___.

We could think of it as the two digit number would be the factor that affects the three digit number, so you want to minimize the two digit number. The rest of the digits could fill up the three digit number. So, you'd want to put the two digit number as 10 and the three digit number as 234.
You end up with 10×234 and the product of those two are 2340.

Problem 8: What is the greatest three-digit number divisible by both 7 and 8?

The lowest common denominator of 7 and 8 is 56. Just multiply 56 by an integer until you get a four digit number. Then, subtract that number by 56 to get the greatest 3 digit number. To get a four digit number, I'd have to multiply 56 by 18 to get 1008. Then, I subtract 56 from it, getting 952.

Problem 9: How many three-digit numbers have a tens digit that is 5, 6, or 9?

Three digit numbers have an interval of 100 ¡ x ¡ 999. For every hundred, there are 10 numbers that have 5 as their tens place. This goes for 6 and 9 as well. In total, there are 30 numbers per hundred that have those numbers in their tens place.

There are 9 hundreds that exist in three digit numbers. 9×30 = 270 numbers.

Problem 10: How many two-digit numbers have the property that their tens digit is less than their ones digit?

There are 89 two digit numbers. In between 10 - 19, 12 to 19 fit this criterion. There are 8 numbers in this row. In between 20 - 29, 23 - 29 fit this criterion. There are 7 numbers in this row.

Take the sum of all the ones in this pattern, and you end up with 8 + 7 + ... + 2 + 1. Add these numbers up and you will get 36 numbers.

Problem 11: A number is called a Niven number if it is divisible by the sum of its digits. How many Niven numbers are there between 1 and 10 inclusive?

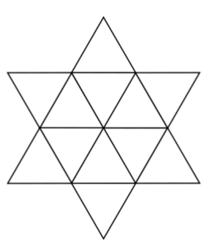
In fact, all of these numbers in this interval are Niven numbers because they can be divisible by the sum of its digits. There are 10 Niven numbers in this interval.

Problem 12: A Marsenne prime is a prime number of the form $2^{n} - 1$ for some integer n. In fact, the largest known prime number $2^{82,589,933}$ - 1 is a Mersenne prime. How many Mersenne primes are there below 100?

Let's look at all of the possibilities that fit this situation.
N = 1 : 2^1 - 1 = 1 (1 is not a prime number)
N = 2 : 2^2 - 1 = 3 (PRIME)
N = 3 : 2^3 - 1 = 7 (PRIME)
N = 4 : 2^4 - 1 = 15 (15 is not prime)
N = 5 : 2^5 - 1 = 31 (PRIME)
N = 6 : 2^6 - 1 = 63 (63 is not a prime number)
N = 7 : 2^7 - 1 = 127 (prime but greater than 100)

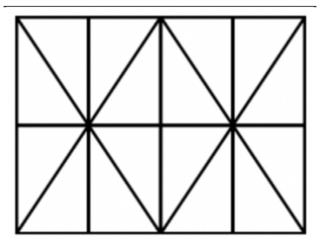
Therefore, there are 3 Mersenne Primes below 100.

Problem 13: How many triangles of any size are in the figure shown?



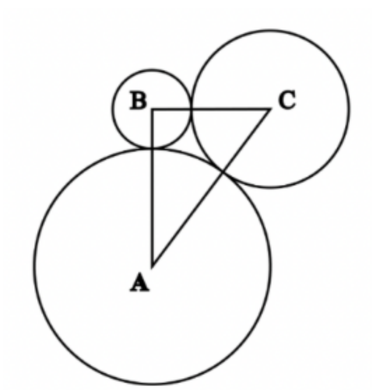
First, look at the triangles that are unit triangles, the triangles by itself. there are 12 of those. Next, look at the triangles that consist of 4 small triangles. there are 6 of those. Then, look at the triangles that consist of 9 triangles (the largest ones). There are 2 of those. In total, there are 20 triangles.

Problem 14: How many triangles of any size are in the figure shown?



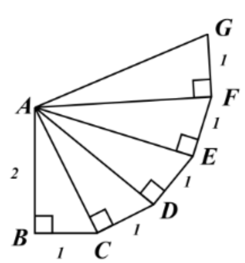
First, take the smallest triangles. There are 16 of those triangles. The next size triangles are the two smallest triangles that are side by side together. There are 10 of those. Then, there are triangles that are made of 4 smallest triangles. There are 8 of those. Finally, there are triangles that are made of 8 triangles, the biggest triangle. There are two of those. In total, there are 36 triangles.

Problem 15: Circle A has a radius of 6, circle B has a radius of 2, and circle C has a radius of 4. They are tangent to each other as shown, and m∠ABC = 90°. What is the area of the triangle whose vertices are the centers of the circles A, B, and C?



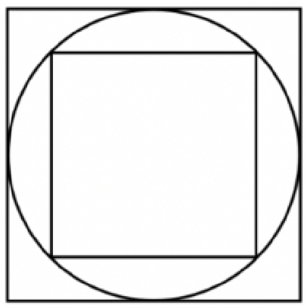
To find the area of the triangle, just find the side lengths, which is BC and AB. BC is the radius of B plus the radius of C. That is 2 + 4 = 6. BC = 6. To find AB, it is the sum of the radius of A and the radius of B. That is 6 + 2 = 8. AB = 8.
The area of a triangle is 1/2 b×h.
Plug the values into the equation and you get 1/2 (6)(8) = 24.

Problem 16: What is the length of AG?



Use the pythagorean theorem : $a^2 + b^2 = c^2$. a and b are the legs of the triangle and c is the hypotenuse of the triangle. You want to continuously find the hypotenuse of the triangle to get to AG. AC is the hypotenuse of △ABC, and would be the leg of △ACD.
$2^2 + 1^2 = c^2$, $c = \sqrt{5}$. Do the same for △ACD and you get $c = \sqrt{6}$. If you see the pattern, it is adding 1 inside the square root every time you go up a triangle.
By the time you get to AG, the hypotenuse of that triangle is $\sqrt{9}$, or 3. So, AG = 3.

Problem 17: A circle is inscribed in a large square and circumscribed about a smaller square. The area of the larger square is 6 square meters. What is the area of the smaller square?

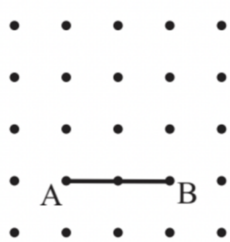


If the area of the larger square is 6 square meters, then the side lengths of that square is $\sqrt{6}$ meters. Because that is the side length, the radius of the circle is half of it, $\sqrt{6}/2$ meters. The furthest distance away from the center of the square to the edge is the radius of the circle. If you use your 45 degree special right triangle, we can see that half of the side length in the smaller square is just $\sqrt{3}/2$ meters. The full side length of the smaller square is $\sqrt{3}$ meters. The area is just the side length squared, so the area is 3 square meters.

Problem 18: How many different squares can be formed by using four of the evenly-spaced dots below as vertices of the square?

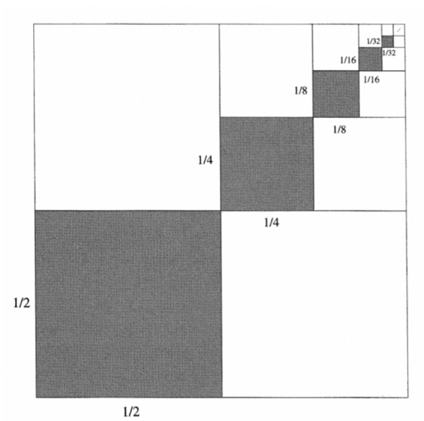
For the squares that only take up one square unit, there are six of those. For the squares that take up four square units, there are two of those. That combines to be 8 different squares.

Problem 19: The dots are evenly spaced vertically and horizontally. Segment AB is drawn using two points, as shown. Point C is to be chosen from the remaining 23 points. How many of these 23 points will result in an isosceles triangle ABC?



Let's place the third point and label it as C. If we place C anywhere on the line of symmetry, we would get an isosceles triangle. Then, since AB has a length of 2, if AC or BC is equivalent to 2, that creates another isosceles triangle. That results in 6 isosceles triangles.

Problem 20: Consider the diagram below. It depicts a square with an area of 1. What is $\sum_{n=1}^{\infty} (\frac{1}{4})^n = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots$? (Hint: tilt your head to the right.)



As you sum up all those values, you realize that you get one third of the entire square. It is an interesting property that works for other fractions too. For example, $\sum_{n=1}^{\infty} (\frac{1}{7})^n$ is equal to 1/6.