CVJMT 2023

Elementary School Problems and Solutions

These are the solutions to the elementary school problems for CVJMT [1] 2023. These solutions include the following in this order : Mental Math, Paper Math, Geometry & Number Theory, and finally, Algebra & Probability.

Mental Math was included in CVJMT [1] because it is an essential skill for everyday life and enables quick and accurate calculations, making it a valuable tool for solving common problems in a time-efficient manner. During the tournament, 20 mental math problems were issued with a time limit of 15 minutes to solve.

Paper Math is included in CVJMT because, like mental math, paper math is an essential part of every-day life. In many real world situations, mathematical problems can be complex and require multiple steps to solve. Paper math allows for a more organized and step-by-step approach, where intermediate results can be recorded and used in subsequent calculations. Additionally, in many fields, such as science and engineering, paper math is used to communicate complex mathematical concepts and results. Equations and calculations are written down on paper and used to explain and justify scientific discoveries and technological advancements. In CVJMT, we issued 20 paper math problems with 30 minutes to complete.

Geometry and Number Theory were categories in CVJMT because these two branches of mathematics are both interesting and vital to higher-level math courses. Despite focusing on different types of mathematical objects, geometry and number theory share a fundamental approach to mathematical reasoning and the study of mathematical structures. Both fields require abstract thinking, logical reasoning, and an understanding of mathematical structures, as well as their practical applications. In the tournament, 20 Geometry and Number Theory questions were offered with a time limit of 15 minutes.

Algebra and Probability were categories in CVJMT because these two branches of mathematics are both indispensable to find unknown quantities. Algebra and Probability both require extensive skills in manipulating equations and formulas. It tests your skills of analytical thinking and problem solving. Ultimately, Algebra and probability are both used in many other fields of mathematics and science. For example, algebra is used in calculus, physics, and engineering, while probability is used in statistics, economics, and computer science. In the tournament, 20 Algebra and Probability questions were prompted with a time limit of 15 minutes.

[1] CVJMT - Castro Valley Junior Math Tournament

Problem 1: What is 1.23 + 45.678?

An easy way to do this in your head is to add the places together (tens place adds with other tens place). You should start with the smallest place. 45.678 has the smallest place, the thousandths place. 1.23 does not have a thousandths place. $8 + 0 = 8$. Move on to the next places. You end with 46.908.

Problem 2: How many seconds does it take to cross a 100 meter bridge if you walk at 50 meters per second?

You can use the distance-speed formula : $distance = speed \times time$
Plugging it into the equation : $100 = 50 \times time$, $time$ = 2 seconds

---

Problem 3: What is x if $2^x = 64$?

2 to the power of a number is 64. A power means that you are multiplying by itself x times. If you multiply 2 by itself 6 times, then you result with 64. Therefore, $\underline{x = 6}$.

Problem 4: What is 5% of 200?

5% is equivalent to 0.05. Multiplying 0.05 and 200 together results in 10

Problem 5: If one egg has enough yolk for two cakes, then how many eggs must be cracked for 5 cakes?

Conversion factor : 1 egg = 2 $cakes$
$5\ cakes \times \frac{1\ egg}{2\ cakes}$ = 2.5 eggs. Since you cannot have half of an egg, the answer is $\underline{3\ eggs}$.

Problem 6: How much money is 7 hundred-dollar bills, 6 ten-dollar bills, 5 one-dollar bills, 4 quarters, 3 dimes, 2 nickels, and 1 penny?

Hundred dollar bills represent $100. Ten dollar bills represent $10. One dollar bills represent $1. Quarters represent $0.25. Dimes represent $0.10. Nickels represent $0.05. Pennies represent $0.01.

Then, sum the numbers all up. $700 + 60 + 5 + 1 + 0.30 + 0.10 + 0.01$ = $766.41

Problem 7: What is 23 + 34 + 42?

Simply add the tens place and then add the ones place.
The tens place would be : $2 + 3 + 4 = 9$.
The ones place would be $3 + 4 + 2 = 9$.
The combined number would be 99.

Problem 8: What is 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10?

You can use a math trick for this problem. Simply add the top and bottom numbers together. For example, 1 + 10 = 11, 2 + 9 = 11, and so on. You can see that they all equal 11. These can be called 'summation pairs'. We can see that there is 5 pairs in this series. 5×11 = 55

Adding them manually would also get you 55.

Problem 9: What is a + b if a = b and b = -4?

If a = b, then a would also equal -4. Therefore, the answer would be $(-4) + (-4)$ = -8

Problem 10: If $\frac{1000}{100+x} = 1$, what is the value of x?

For the fraction to equal to 1, the numerator (top portion) must be equal to the denominator (bottom portion). So, $1000 = 100 + x$.

Solving for x, we find that $x = 900$.

Problem 11: Simplify $(-2x) \times 7 - 10x$.

By the rule of *PEMDAS*, we multiply before subtraction. $-2x \times 7 = -14x$.
Next, $-14x - 10x = -24x$

Problem 12: What is the smallest palindrome greater than 1392? A palindrome is a number that reads the same forwards or backwards, such as 3443.

Since there are four digits in 1392, that means that the palindrome really depends on the first two digits. If we are trying to go for the smallest palindrome, we could try 1331, but that number is not bigger than 1392. We can add one to the set of two digits. We end up with 1441. That is bigger than 1392. 1441.

Problem 13: How many numbers in the set below are odd numbers?
$\{11, 12, 108, 115, 117, 289\}$

Odd numbers are numbers that end with $1, 3, 5, 7$, and 9. 11, 115, 117, and 289 meet that criterion. Therefore, there are 4 odd numbers.

Problem 14: How many numbers in the set below are divisible by 5?
$\{32, 10, 15, 18, 52, 30\}$

Numbers that are divisible by 5 are numbers that end with 0 or 5. 10, 15, and 30 fit that criterion. Therefore, there are 3 numbers divisible by 5.

Problem 15: How many numbers in the set below are divisible by 3?
$\{11, 33, 128, 1566, 378, 9\}$

There is no complete trick for numbers that are divisible by 3. We can look at each number individually. We know that 11 is not divisible by 3. 9 is divisible by 3. So is 33. You can tell that 128 is not. 1566 is because 15 is divisible and 66 is also divisible. 378 is also divisible because 3 is divisible and 78 is too.
The trick is taking two numbers at the same time and dividing by three. For example, for 1566, take 66 and take 15. If they are both divisible by 3, then 1566 is divisible by 3. Therefore, there are 4 numbers in this set that are divisible by 3.

Problem 16: Which two numbers have a sum of 14 and a product of 45?

By intuition, you can tell that the answer is 9 and 5.

However, you can solve with algebra. Set the two nunmbers to x and y.

$$x + y = 14 \qquad \text{(Sum)}$$
$$x \times y = 45 \qquad \text{(Product)}$$

$$y = 14 - x \qquad \text{(from Sum)}$$
$$x \times (14 - x) = 45 \qquad \text{(Plug into equation)}$$
$$14x - x^2 = 45$$
$$x^2 - 14x + 45 = 0$$
$$(x - 9)(x - 5) = 0 \qquad \text{(Or use quadratic equation)}$$
$$\underline{x = 9, 5}$$

Problem 17: If $x^x = 4$, what is the value of x?

x = 2 is the only result that works.

Problem 18: Mark sets off at 7 : 40 AM, and his journey takes 45 minutes. At what time does he arrive?

Add 45 minutes to the minute portion of the time. You result with : 7 : 85 AM. Since the minute hand cannot go past 60, you subtract 60 and add an hour to the hour portion. Your result is 8 : 25 AM.

Problem 19: Kyle reads 24 pages a day. *The Great Gatsby* has 264 pages. How many days does it take for him to finish it?

You can tell by intuition that if you divide the total number of pages by the amount of pages Kyle reads per day, you can get the number of days. 11 days.

Conversion factor : 24 pages = 1 day. 264 pages $\times \frac{1 \text{ day}}{24 \text{ pages}}$ = 11 days.

Problem 20: What is the largest area that can be enclosed within 24 feet of fencing?

You may think that the largest area is a square, but it is a circle. Since the circumference (fencing) is 24 feet, and the circumference of a circle is $2\pi r$, $r = 12/\pi$.

Now, the area of a circle is πr^2. Plugging r into the area equation, we result in $\frac{144}{\pi}$.

Problem 1: Evaluate $5 \times 4 + 15 - 10$.

By the *PEMDAS*[1] Rule, you multiply first, then add and subtract next. Then, simplify.

$$\begin{aligned} 5 \times 4 + 15 - 10 &= ? \\ 20 + 15 - 10 &= ? \ (multiply) \\ 35 - 10 &= ? \ (add) \\ 25 &= ? \ (subtract) \end{aligned}$$

Therefore, the answer is 25

Problem 2: While shopping for party supplies, Bessie the Cow sees a group of cows and chickens that she can invite to her Pi Day Party. In total, Bessie counts 3 heads and 8 feet. How many chickens are there?

- - - - - - -

Given :
3 heads and 8 feet

- - - - - - -

What we can infer :
Cows have 1 head and 4 feet
Chicken have 1 head and 2 feet
Since there are three heads, there are three animals.

- - - - - - -

There are many ways to do this problem, but one of them are shown.

You can think in your head, if there are three animals, the possible combinations could be : 0 cows, 3 chickens. It can also be 1 cow, 2 chickens. Another alternative would be 2 cows and 1 chicken. Finally, it could be 3 cows and no chicken.

Let's try each one :

$$\begin{aligned} 0 \text{ cows and } 3 \text{ chicken} &: 3 \text{ heads and } 6 \text{ feet} \\ \underline{1 \text{ cow and } 2 \text{ chicken}} &: \underline{3 \text{ heads and } 8 \text{ feet}} \\ 2 \text{ cows and } 1 \text{ chicken} &: 3 \text{ heads and } 10 \text{ feet} \\ 3 \text{ cows and } 0 \text{ chicken} &: 3 \text{ heads and } 12 \text{ feet} \end{aligned}$$

We can then see from analysis that the correct answer is 1 cow and 2 chicken.

---

[1]PEMDAS - A guideline of what steps to do first. The list, from first to last : Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction.

Problem 3: Becky the Cow wants to invite 2 times as many cows as chickens and 3 times as many chickens as sheep. Becky invites 2 sheep to the party. How many cows should she invite?

Dimensional Analysis [1] is the way to go for this problem.

Conversion factors are listed :

- 2 cows = 1 chicken
- 3 chicken = 1 sheep

2 sheep $\times \frac{3\ chicken}{1\ sheep} \times \frac{2\ cows}{1\ chicken}$ = 12 cows.

Problem 4: Bella the Cow wants to build tables for the party. She doesn't have a ruler, but she knows that the length of 4 pigs standing in a line equals 7 meters. If she wants to make a table that is 20 pigs long and 12 pigs wide, what is the area of the table in square meters?

Dimensional Analysis is the way to go for this problem.

Conversion factors are listed :

- 4 pigs = 7 meters

Area is equal (20 $\times$ 12) $pigs^2$ = 240 $pigs^2$.

240 $pigs^2 \times \frac{7\ meters}{4\ pigs} \times \frac{7\ meters}{4\ pigs}$ = 735 $meters^2$.

Problem 5: Abby the Cow brought a group of pigs to play pie roulette. There are 35 pigs total: 15 have brown spots, 4 have black spots, and the rest have no spots. Assuming that every pig has an equal chance of being pied, what is the probability that the pied pig has no spots?

Given :

- 15 pigs with brown spots
- 4 pigs with black spots
- 35 - 15 - 4 = 16 pigs with no spots

Probability is equal to amount of pigs that are waiting to be chosen over the total amount of pigs.

$\frac{16\ pigs\ with\ no\ spots}{35\ total\ pigs} = \frac{16}{35}$.

---

[1] A way of manipulating (converting) numbers to get to your answer

---

Problem 6: Bobby the Cow is an extremely picky eater. He wants his pie slice to be 5/7 of the total pie. If the pie's radius is 7 inches long, what is the area of the pie slice that Bobby gets? Leave your answer in terms of π.

The area of the entire pie is πr^2 where r = 7. $\pi \times (7)^2 = 49\pi$.

Since you are trying to solve for 5/7 of the area, you can just multiply the entire area by 5/7.

$5/7 \times 49\pi = \underline{35\pi}$.

Problem 7: Alice the Cow has many hobbies. One of them is making soccer balls. She has 490 pentagonal panels and 921 hexagonal panels. Each ball needs 12 pentagonal panels and 20 hexagonal panels. How many soccer balls can she make?

Since each ball requires 12 pentagonal panels and 20 hexagonal panels, you must have those 'ingredients' to make a ball, no less, otherwise you cannot assemble a ball. A good way to see it is to see how many balls you can make using just pentagonal panels or just hexagonal panels.

490 pentagonal panels $\times \frac{1 \text{ ball}}{12 \text{ pentagonal panels}}$ = 40.833 balls

921 hexagonal panels $\times \frac{1 \text{ ball}}{20 \text{ hexagonal panels}}$ = 46.05 balls

* During the test, you just need the approximate values to see which is the limiting factor.

We can see from the data above that the pentagonal panels are the limiting factor. You can only make <u>40 total balls</u>.

Problem 8: Alice the Cow also likes selling apples. If an apple sells for \$2.50, and an Apple computer costs \$1000, how many apples will Alice need to sell to buy an Apple computer?

Conversion factors are listed:

- 1 apple = \$2.50
- 1 apple computer = \$1000.00

Dimensional Analysis is the way to go!
1 apple computer $\times \frac{\$1000}{1 \text{ apple computer}} \times \frac{1 \text{ apple}}{\$2.50}$ = <u>400 apples</u>.

Problem 9: Farmer Mark recently bought a rectangular plot of land. It has an area of 60 square feet, and it is 15 feet wide. If he wants to build a fence around the perimeter, how many feet of fencing does he need?

Using the area and one of the sides, we can solve for the other side.

Area = length $\times$ width
60 = 15 $\times$ length
length = 4 feet
Now that we know the length is 4 feet and the width is 15 feet, we can solve for the perimeter. The formula for perimeter is : (Perimeter = 2 $\times$ length + 2 $\times$ width)

$2 \times 15 + 2 \times 4 = 30 + 8 = \underline{38\ \text{ft}^2}$.

Problem 10: Farmer Isabella plans to steal Farmer Alex's chickens. Isabella knows that Alex has 5 chicken coops and 45 chickens. Every chicken coop has at least one chicken, but Isabella doesn't know exactly how many chickens there are in each chicken coop. She only has time to visit two chicken coops. What is the maximum number of chickens that could be in two coops?

If there are at least 1 chicken in each coop for the 5 coops, we can maximize choosing two coops by having only 1 chicken in the other coops and having most of the chicken in one of the coops.

So, there are 1 chicken in each of 4 coops. There are then 41 chicken in the remaining coop. Choosing two coops would get you 42 chicken.

Problem 11: Farmer Emily forgot her four-digit PIN! She knows the following information: the first digit is A, D, or E; the second digit is 3 or 4; the third digit is 5 or 6; and the last digit is 0. What is the maximum number of PINs she has to try?

A good way to solve this problem is to find out the total amount of combinations is multiplying the amount of combinations in each subsection (each digit in this case).

The first digit has three combinations. The second digit has 2 combinations. The third digit has two combinations. The last digit only has one combination.

Multiplying those combinations together can get the total amount of combinations.

Total combinations : $3 \times 2 \times 2 \times 1$ = 12 combinations.

Problem 12: The four-digit number 567X is divisible by 5, and the digit X is greater than 4. What is the value of X?

A number is divisible by 5 if it ends with either 5 or 0. Since the digit 'X' is greater than 4, the value of X must be 5.

Problem 13: If Alex jogs 1.5 miles every day, how many miles will he jog in 2 weeks?

Dimensional Analysis is the way to go for this problem.

Conversion factors are listed :
- 1 week = 7 days
- 1.5 miles = 1 day

2 weeks $\frac{7\ days}{1\ week} \times \frac{1.5\ miles}{1\ day}$ = 21 miles

---

Problem 14: Chloe wants to drive to LA from Castro Valley. It is 350 miles from LA to Castro Valley. Halfway through, she realized she forgot something at home, and she drove back to get it. She decides that it was not worth driving back to LA again. How many miles has she driven?

She drove halfway there and back. This is equivalent to driving all the way there once. So, the answer is 350 miles.

Problem 15: If 1760 yards are in a mile, how many miles are in a yard?

Use dimensional analysis for this problem.

Conversion Factors :
- 1760 yards = 1 mile

1 yard $\times \frac{1\ mile}{1760\ yards}$ = 1/1760 miles

Problem 16: Soolynn and Joyce are in a heated debate about their favorite numbers. Soolynn's favorite number is $\pi \approx 3.14159$. Joyce's favorite number is $\tau \approx 6.28318$. Soolynn claims that the product of the first six digits of her favorite number is greater than the product of the first six digits of Joyce's favorite number. Find the positive difference between those two values.

The problem only asks for the difference between the two numbers. τ is just 2π. So, the difference is just π.

Problem 17: Chloe the investor is able to multiply her net worth by 3 every year. If she starts off with $500, how much will she have in 4 years?

If you can multiply your net worth by 3 every year for 4 years, that means that you are multiplying your initial net worth by $3 \times 3 \times 3 \times 3 = 81$. Multiplying 81 by 500 gets you $40500.

Problem 18: Katie is super enthusiastic about the Fibonacci sequence. The first few terms are 1, 1, 2, 3, 5, 8, . . . What is the next term of the Fibonacci sequence?

The fibonacci's sequence works by adding the previous number and the current number to get the next numeber. For example, if you are on the fifth term, add 5 and the previous term, 3, to get the sixth term, 8.

The next term is 13.

Problem 19: If $2^x \times 3^x = 36$, what is the value of $2x + 3x$?

The roots of 36 are 2, 2, 3, and 3. These four numbers multiplied with each other results in 36. In the exponent form, you can see that $x = 2$. You are trying to solve for $2x + 3x$, which is $5x$. If $x = 2$, then the answer is 10.

Problem 20: Erin is on an adventure! She starts at M in the diagram and can either move to an adjacent letter to the right, left, up, or down. How many paths can she make that spell the word MATH?

Split the diagram into a quarter of the diagram.

```
H
T H
A T H
M A T H
```

There is 2^3 permutations just for this quarter. Since there is one path that overlap for each quarter, you subtract 4 from the total permutations. You have 32 - 4 = 28 permutations (paths).

```
      H
    H T H
  H T A T H
H T A M A T H
  H T A T H
    H T H
      H
```

Problem 1: Let's play a game!

1. Pick a number between 1 and 25
2. Add 9 to it
3. Multiply the result by 3
4. Subtract 6
5. Divide by 3
6. Subtract your original number

What number do you have?

Let's say the number that you chose was x. Add 9 to it. You get 'x + 9'.

Then, multiply the result by 3. You get : 3(x + 9) = 3x + 27

Subtract 6. 3x + 27 - 6 = 3x + 21

Divide by 3. $\frac{3x+21}{3}$ = x + 7

Subtract original number (x). x + 7 - x = 7.

Problem 2: If (4 + x) + (5 + x) + (6 + x) = 60, what is the value of x?

Simplify the equation and then solve for x.

$$\begin{aligned} 4 + x + 5 + x + 6 + x &= 60 \\ 3x + 15 &= 60 \\ 3x &= 45 \\ \underline{x} &= \underline{15} \end{aligned}$$

Problem 3: When paper strip a with a length of 23 inches is attached onto paper strip y, the new strip of paper has a length of 31 inches. If the length of the overlapping part is 2 inches, how long is paper strip y?

The equation goes : 23 + y - 2 = 31. The sum of the two lengths minus the overlapping part is the total length (31 inches)
y + 21 = 31, y = 10

Problem 4: Anna can finish 2 pages of her summer homework in 1 hour. How many pages can she finish in 2 hours?

Conversion factors :

- 2 pages = 1 hour

2 hours $\times \frac{2\text{ pages}}{1\text{ hour}}$ = 4 hours

---

Problem 5: According to the poster on a printing shop, it costs $1 to print each page. However, they offer a $20 discount to customers who print more than 500 pages. John is planning to print less than 500 pages, but he realized that the total cost would be cheaper if he printed 500 pages. What is the minimum possible number of pages John is planning to print?

Since there is a discount and you're solving for the mininum number, you want to make use of the discount. The minimum amount of pages that someone needs to print to get the discount is 500 pages. 500 pages costs $480.

So, the minimum amount of pages that he would have printed costs more than the discounted price. $481 is the next number higher than 480. To get to $481, John would have printed 481 pages.

Problem 6: Eva made a sugar solution with 20g of sugar and 100g of water. She then added 5g of sugar into the solution. What fraction of the solution is sugar?

The initial solution has 120g of solution and 20g of sugar. That means that 1/6 of the solution is sugar. After adding the extra 5g of sugar, you would have 25g of sugar in 125g total solution.

The fraction of sugar in solution is 25/125 = 1/5

Problem 7: Car A and Car B are on different ends of the road. Car A is driving toward Car B at 60 mph, and Car B is driving toward Car A at 40 mph. If the cars are 200 miles apart, how many hours will it take for them to meet each other?

The change in distance between the two cars is 100 mph because Car A is going to B at 60 mph and Car B is going to A at 40 mph. 60 + 40 = 100 mph.

Using the distance formula : $d = vt$

Plugging into the equation : $200 = 100 \times t$, $t = 2$.

2 hours

Problem 8: A card is randomly drawn from a standard 52-card deck. What is the probability that it is a red card?

Half of the deck is a red card, so the probability is 1/2 or 50%

Problem 9: There are two times more blue candies than green candies in a bag. What is the probability of picking out a blue candy?

If there is one green candy in the bag, that means that there are two blue candies in the bag. If you randomly choose a candy, there is a 2/3 chance of choosing out a blue candy.

Problem 10: We have two positive integers x and y such that x + y = xy = 4. What is 2x/y?

This is a special case. The only way that x + y = xy = 4 is if x and y are both equal to 2.

Plugging it into the equation, 2x/y = 2(2/2) = 2.

Problem 11: Bill has 4 red socks, 5 pink socks, 9 black socks, and 17 purple socks. What is the minimum number of socks Bill needs to pull from his sock closet to guarantee that he has at least one pair of matching socks?

If he has one pair of each sock, or 4 socks in total, he just needs one more sock until he can guarantee a pair. 5 socks

---

Problem 12: Eric wants to walk to the train station. The distance between his house and the train station is 8 cm on a map with a scale of 1 : 25000. How many meters will Eric have to walk?

Since the real distance of every centimeter on the map is 25000 cm, you can just multiply the map distance by 25000. The answer is going to be 8×25000 = 2,000,000 centimeters. However, the question is asking the distance in meters, so just divide by 1000 to get to meters from centimeters. 2000 meters.

Problem 13: The teacher is calculating the average score of each row in his classroom. In a row where there are 5 students sitting, the first three students' average score is 83, while the last two students' average score is 77. What is the average score of this row?

Since the first three students have an average score of 83, they would have a total of 249 (83×3) points. The next two students have an average score of 77, meaning they have a total of 154 (77×2) points. The total score out of all 5 students is 403 points. Then, take the average score, or the mean of this row.

403/5 = 80.6 points

Problem 14: What is the next number in the sequence 3, 6, 10, 15, 21, ?

You need to figure out the pattern for this problem. The pattern is that the difference between the number and the previous number is increasing 1 every time you shift one over. For example, the difference between the 1st and 2nd number is 3, while the difference between the 2nd and 3rd number is 3 + 1, or 4. Keep on going, and you'll see that the final number is 28.

Problem 15: A bottle of soda costs $20. The soda costs $19 more than the empty bottle. How much does the empty bottle cost?

Let's say the bottle of soda is x. So, x = 20

Let's also say that the empty bottle is y. Since the soda costs 19 dollars more than the empty bottle, we can set up an equation for that.

$x = 19 + y$, $20 = 19 + y$, $y = 1$

Therefore, the empty bottle costs 1 dollar

Problem 16: According to the directions on a bottle of liquid laundry detergent, the amount detergent used each time should be 1/3 of the cap. However, Katie mistakenly read it as 1/2 of the cap. When she realized this, she had already used the detergent 12 times. Following the correct instructions, it took her 15 more times to finish off the bottle of detergent. How much detergent is in the bottle (in caps)?

Katie has used the detergent 12 times using 1/2 of the cap. That means she has used a total of 6 full caps. Then, she corrected it to 1/3 of a cap 15 times. That is 5 whole caps. The sum of the two is 11 total caps.

Problem 17: The plan for a tree planting event was to plant 20 trees per hour. However, during the actual event, 8 more trees were being planted per hour. If the event lasted 2 hours, many trees were planted during the event?

The rate of tree planting shifted from 20 trees an hour to 28 trees an hour because 8 more trees were planted per hour. Since the event lasted 2 hours, they planted 56 trees.

Problem 18: Charlie created a new @ operator. It is defined as a@b = 6a + 5b. What is 20@5?

An @ operator is basically a function where you can plug in the numbers.

In this example, the operator lists : 6a + 5b, where a = 20 and b = 5. Plug it into the equation.

6(20) + 5(5) = 120 + 25 = 145

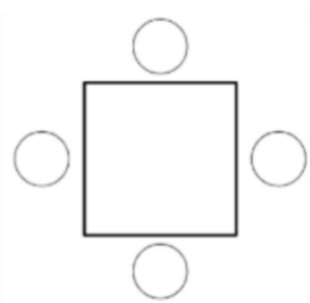
Problem 19: Emily and Thomas have homework. Emily finishes each page in 20 minutes, and Thomas finishes each page in 40 minutes. They decide to work together to finish the 6 total pages. How many minutes will it take for them to finish?

How many pages are done in 40 minutes? Emily can do 2 pages and Thomas can do 1. So, 3 pages are done every 40 minutes. The conversion factor would then be

3 pages = 40 minutes.

6 pages $\times \frac{40 \text{ minutes}}{3 \text{ pages}}$ = 80 minutes.

Problem 20: Laura, her brother, and their parents are dining at a restaurant. The arrangement of their seats is shown in the diagram below. If they pick their seats randomly, what is the probability that Laura sits to the left of her brother?



If they pick seats randomly, you can place Laura at any spot. There is always only one spot on the left of Laura. There are 3 total open spaces. So, the probability that her brother sits next to her is 1/3.

Problem 1: Two angles in a triangle measure 12 degrees and 34 degrees. What is the measure of the third angle?

Triangles have a property that the sum of all of the internal angles are 180 degrees. Let the missing angle be X.

X + 12 + 34 = 180 , X = 180 - 46 , X = 134

Therefore, the measure of the third angle is 134 degrees.

Problem 2: There is a right triangle with legs of length 5 and 12. What is the area of the triangle?

The formula for the area of a triangle is 1/2 base $\times$ height. The base measure is 5 and the height measure is 12.

Plug it into the equation : $1/2 \times 5 \times 12 = \underline{30}$ units^2.

Problem 3: What is the perimeter of a hexagon with side lengths of 4, 5, 8, 4, 5, and 8?

The perimeter of a hexagon would be all of the side lengths added up together. That would be 4 + 5 + 8 + 4 + 5 + 8. That equals 34.

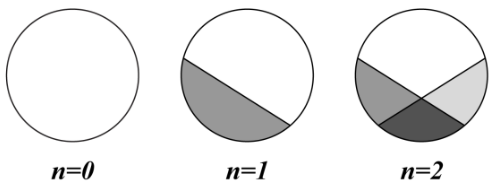
Problem 4: How many lines of symmetry are in an equilateral triangle?

Lines of symmetry are lines that go through the shape directly and splits the shape to two identical pieces. Construct an equiliateral triangle. You get a line of symmetry if you slice it down straight in the middle. Then, rotate the triangle around a little, and slice it down again. You end up with 3 lines of symmetry.

Problem 5: What is the name of the polygon that has 5 sides?

The names of polygons follow the Latin Prefixes. Tri = 3, Penta = 5, Hexa = 6, Hepta = 7, Octa = 8, and so on. The polygon that has 5 sides is called a Pentagon.

Problem 6: What is the maximum number of pizza slices that can be made with only 3 straight cuts? An example with 0, 1, and 2 cuts is shown below.



The goal is to make the most slices with 3 cuts. The first two cuts are already shown. The strategy is to cut down straight through most of the regions, but not cutting through an intersection point. To picture this, you can cut straight down through a little off to the side of the middle. That would get you a total of 7 slices.

Problem 7: What is the smallest number that is divisible by both 4 and 5?

A number that is both divisible by 4 and 5 is a number that is divisible by 20, since 20 is the least common multiple of 4 and 5. The smallest number that first this criteria is 20.

Problem 8: What is the smallest possible product of a two-digit number and a three-digit number obtained from five distinct digits?

Since you are aiming for the smallest numbers, you want the smallest digits. Those digits are 0, 1, 2, 3, and 4. Since you can't put 0 as your first digit of a number, it is most logical to put it as the second digit of the three digit number. So you are now left with _0_ $\times$ __.

To minimize the product, you want the biggest numbers in the ones place. Let's put 4 in the ones place of the three digit number. Also, you want the smallest numbers in the hundreds place, so let's put 1 in the hundreds place in the three digit number. You are left with 104 $\times$ __.

Since you have 2 and 3 left, it is most logical to put the 2 before the 3 because the 2 is smaller than the 3.

You end up with 104 $\times$ 23 and the produc of those two are 2392.

Problem 9: How many three-digit numbers have a tens digit that is 5, 6, or 9?

Three digit numbers have an interval of 100 ¡ x ¡ 999. For every hundred, there are 10 numbers that have 5 as their tens place. This goes for 6 and 9 as well. In total, there are 30 numbers per hundred that have those numbers in their tens place.

There are 9 hundreds that exist in three digit numbers. 9 $\times$ 30 = 270 numbers.

Problem 10: There is a rectangle with a side length of 9 inches. The perimeter is 42 inches. What is the area of the rectangle?

Since the perimeter is 42 inches, two side lengths plus two side widths equal 42. Let the widths be y and the lengths be x. 2x + 2y = 42. Since x = 9, 18 + 2y = 42.

2y = 42 - 18 , 2y = 24 , y = 12

Now that we know all of the side lengths, we can solve for the area. The area of the rectangle is just length $\times$ width. Plugging the values into the equation,
Area = 12 $\times$ 9 = 108 inches^2.

Problem 11: How many rectangles have whole number side lengths and an area of 10? Note that an m $\times$ n rectangle is distinct from an n $\times$ m rectangle.

The number of rectangles that have integer (whole numbers) side lengths with an area of 10 is equivalent to the number of integer factors that 10 has. The factors of 10 are 1, 2, 5, and 10. 10 has 4 factors. There are 4 such rectangles.

This is true because the posible rectangles that exist are 1x10, 2x5, 5x2, and 10x1.

Problem 12: How many factors do prime numbers have?

The definition of prime numbers are that it doesn't have any other factors besides the factor of themselves multiplied by 1. So, the only factors are themselves and 1, which sum up to 2 factors.

Problem 13: How many prime numbers are there between 1 and 13 inclusive?

The prime numbers that in between 1 and 13 are 2, 3, 5, 7, 11, and 13. Prime numbers are numbers that don't have any other factors besides themselves and 1. There are 6 prime numbers in this interval.

Problem 14: A number is called a Niven number if it is divisible by the sum of its digits. How many Niven numbers are there between 1 and 10 inclusive?

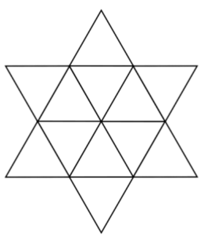
All of the numbers are Niven numbers in the interval of (1,10). All of the single digit numbers are prime numbers because you're dividing by itself, and you can always do that. Therefore, there are 10 numbers that fit this criteria.

Problem 15: A Marsenne prime is a prime number of the form $2^{n} - 1$ for some integer n. In fact, the largest known prime number $2^{82,589,933}$ - 1 is a Mersenne prime. How many Mersenne primes are there below 100?

Let's look at all of the possibilities that fit this situation.
N = 1 : 2^1 - 1 = 1 (1 is not a prime number)
N = 2 : 2^2 - 1 = 3 (PRIME)
N = 3 : 2^3 - 1 = 7 (PRIME)
N = 4 : 2^4 - 1 = 15 (15 is not prime)
N = 5 : 2^5 - 1 = 31 (PRIME)
N = 6 : 2^6 - 1 = 63 (63 is not a prime number)
N = 7 : 2^7 - 1 = 127 (prime but greater than 100)

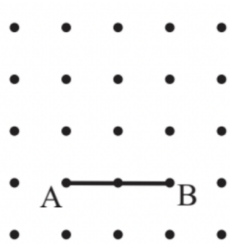
Therefore, there are 3 Mersenne Primes below 100.

Problem 16: How many triangles of any size are in the figure shown?



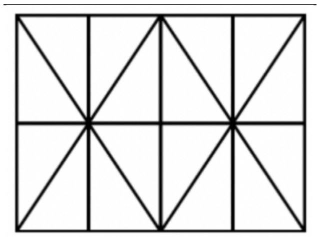
First, look at the triangles that are unit triangles, the triangles by itself. there are 12 of those. Next, look at the triangles that consist of 4 small triangles. there are 6 of those. Then, look at the triangles that consist of 9 triangles (the largest ones). There are 2 of those. In total, there are 20 triangles.

Problem 17: The dots are evenly spaced vertically and horizontally. Segment AB is drawn using two points, as shown. Point C is to be chosen from the remaining 23 points. How many of these 23 points will result in an isosceles triangle ABC?



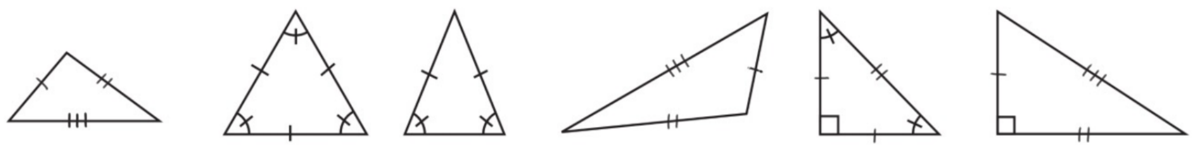
Let's place the third point and label it as C. If we place C anywhere on the line of symmetry, we would get an isosceles triangle. Then, since AB has a length of 2, if AC or BC is equivalent to 2, that creates another isosceles triangle. That results in 6 isosceles triangles.

Problem 18: How many triangles of any size are in the figure shown?



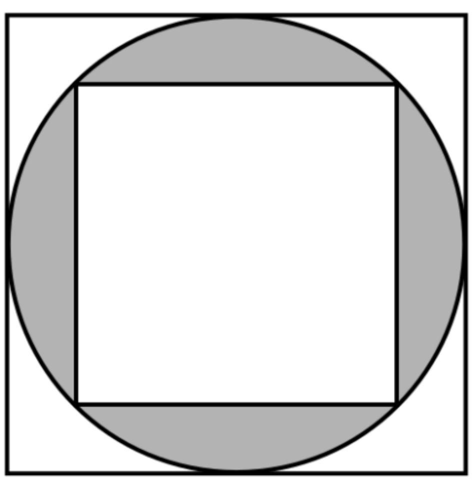
First, take the smallest triangles. There are 16 of those triangles. The next size triangles are the two smallest triangles that are side by side together. There are 10 of those. Then, there are triangles that are made of 4 smallest triangles. There are 8 of those. Finally, there are triangles that are made of 8 triangles, the biggest triangle. There are two of those. In total, there are 36 triangles.

Problem 19: How many of these triangles are scalene triangles?



Scalene triangles are triangles that do not have any side lengths with the same length. The first triangle is an example of a scalene triangle. There are a total of 3 scalene triangles.

Problem 20: A circle is inscribed in a large square and circumscribed about a smaller square. The area of the larger square is 9 and the area of the smaller square is 4.5. What is the area of the shaded region?



Since the area of the larger square is 9, then the side lengths of the larger square is 3. The side length of the larger square is also equivalent to the diameter. So the radius would be 1.5. The area of a circle is πr^2. The area of the circle is 2.25π.

The shaded area is the area of the circle minus the area of the smaller square. The total area is then computed to be $2.25\pi - 4.5$.