# 2022 Castro Valley Junior Math Tournament Problems

**Read the tournament details (https://www.cvhsmath.org/cvjmt/details/) and the rules (https://www.cvhsmath.org/cvjmt/rules/) before you begin. Unless otherwise specified, diagrams are not necessarily drawn to scale. Submissions are due at midnight on March 26th. Send questions to cvmualphatheta@gmail.com.**

1. 150 Pteranodons have gathered on the beach to trade Dinomon cards. Each Pterandon brought one card. 85 Pterandons brought shiny Dinomon cards, and the rest brought regular cards. If Megalodon Maddie randomly chooses a Pteranodon to trade with, what is the probability that they didn't bring a shiny Dinomon card? Express your answer as a **simplified** fraction.

2. Emily is making soccer balls. Each ball needs 12 pentagonal panels and 20 hexagonal panels. If she has 490 pentagonal panels and 921 hexagonal panels, how many soccer balls can she make?

3. A dinosaur can eat 6 trees in 4.2 days. How many days does it take for this dinosaur to eat 10 trees at the same rate? Express your answer as a decimal rounded to the nearest tenth if necessary.

4. Find the value of $x$. Express your answer as a decimal rounded to the nearest integer if necessary.
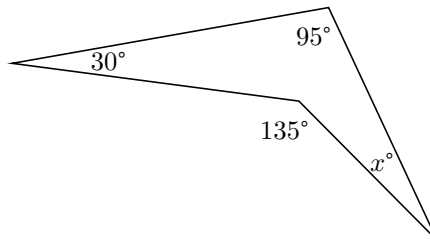


5. Two positive variables are inversely proportional. If one is increased by 42%, what is the percentage change of the other variable? Enter a negative value if the change is a decrease. Round your answer to the nearest tenth of a percent.

6. Cedric applied to 5 colleges. He has an 80% chance of getting accepted by college A, a 5% chance of getting accepted by college B, a 20% chance of getting accepted by college C, a 90% chance of getting accepted by college D, and a 70% chance of getting accepted by college E. What is the expected value of the number of colleges that Cedric will get into? Express your answer as a decimal rounded to the nearest tenth if necessary.

7. A rock has a density of 10 grams per cubic centimeter. If another rock has the same shape but is 3 times as tall and 3 times heavier, what is its density in grams per cubic centimeter? Express your answer as a decimal rounded to the nearest hundredth if necessary.

8. Find the area of the shape below formed by 90° and 180° arcs from circles of radius 1. Express your answer as a decimal rounded to the nearest hundredth if necessary.
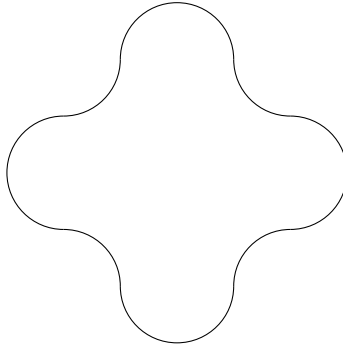
9. Aidan has a cylinder of radius 3 with a slanted top face. The vertical distances from the bottom to the lowest and highest points of the top face are 10 and 15 respectively. What is the volume of this solid? Express your answer as a decimal rounded to the nearest tenth if necessary.
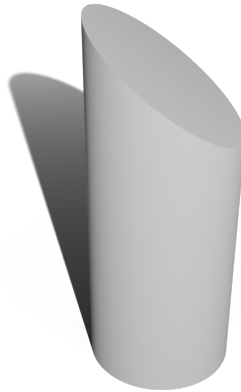
10. Henry and Stella are playing a weird game. In each round, Stella writes down either "heads" or "tails." Henry then flips a fair coin. If the coin flip matches Stella's prediction, she gets a point. Otherwise, Henry gets a point. They play until one of them gets 256 points. What is the probability of Henry getting to 256 points first? Express your answer as a decimal rounded to the nearest hundredth if necessary.

11. Barry the dinosaur is currently 15 meters away from the edge of a circular lava pool with a diameter of 8 meters. If the pool is expanding from the center and its area is increasing at 0.1 square meters per second, how long will it take for the edge of the pool to reach Barry's current position? Express your answer in seconds rounded to the nearest integer if necessary.

12. Vicky has three different pairs of shoes. In how many ways can she arrange the six shoes in a line such that the two shoes in each pair are either next to each other or separated by one shoe from another pair? The order of the two shoes in each pair matters.

13. How many different grids can be obtained by performing zero or more operations to the following grid, where each operation either swaps two rows or two columns?

| 1 | 2 | 3 |
|---|---|---|
| 6 | 5 | 4 |
| 7 | 8 | 9 |

14. Sean randomly and uniformly chooses two different faces of an elongated square gyrobicupola. What is the probability that the two faces share an edge? Express your answer as a **simplified** fraction.

15. Alec's scores on the last 3 tests are 97, 93, and 88. If we assume Alec's score on the next test is a uniform random **integer** from 80 to 100 inclusive, what is the probability that the average score on the four tests is at least 90? Express your answer as a **simplified** fraction.

16. Barry the Brontosaurus was walking from the blueberry bush to the begonia bush when he saw an meteor heading straight for his head! At that instant, the meteor was 40 meters directly above the begonia bush. Barry was 16 meters tall, so he ducked his head before the meteor hit him, and it hit the blueberry bush instead. If he was 37 meters away from the begonia bush, what was the distance in meters between him and the blueberry bush? The meteor is moving extremely quickly, so assume its trajectory is a straight line. Also assume that Barry is a vertical line. Express your answer as a decimal rounded to the nearest tenth if necessary.
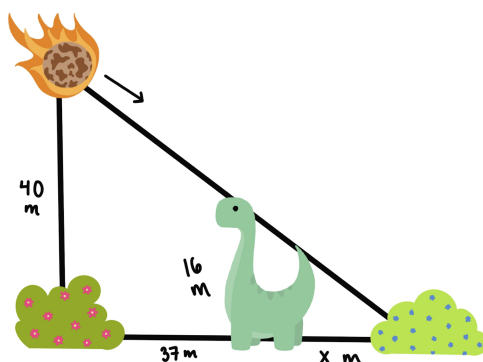


17. In how many ways can Chloe form a committee of 12 dinosaurs out of a total of 10 green dinosaurs and 8 brown dinosaurs if the number of green dinosaurs in the committee must be at least twice the number of brown dinosaurs in the committee?

18. Andrew flips a fair coin. If the result is heads, he moves two meters to the right. If the result is tails, he moves one meter to the left. He repeats this five times. When he is done, what is the probability that he will be at least six meters to the right of his starting point? Express your answer as a **simplified** fraction.

19. $k$ is an unknown positive integer and $\gcd(k, 360) = x$. If $k$ is chosen such that $x \geq 75$, what is the minimum possible value of $x$?

20. Julia rolls a standard six-sided die. If the result is less than 3, she rolls the die a second time. Otherwise, she rolls a die where two of the faces have 2 dots and the four other faces have 4 dots. She multiplies the results of the two rolls together. What is the expected value of this product? Express your answer as a decimal rounded to the nearest hundredth if necessary.

21. A dinosaur population increases exponentially and triples every seven days. In a fourteen-day period, the population increased by 72 dinosaurs. What was the number of dinosaurs at the start of the period?

22. Daniel dug a big hole in the ground with a radius of $75\,\text{cm}$ and covered it with a ball that has a radius of $1\,\text{m}$. What is the vertical distance in centimeters from the ground to the top of the ball? Express your answer as a decimal rounded to the nearest tenth.

23. If Alex's scores on the next two tests are independent and uniform random **real numbers** from 60 to 100, what is the probability of his average score on the two tests being at least 90? Express your answer as a **simplified** fraction.

24. In the figure below, point $O$ is the center of the circle and $\overline{AC}$ is tangent to the circle. If $m\angle ACD = 28°$, what is the value of $m\angle ABD$ in degrees? Express your answer as a decimal rounded to the nearest tenth if necessary.
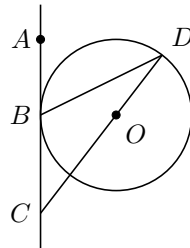


25. The layout of Dinocity is based on a grid. In the following map, the lines represent streets, all of which are one-way streets that either go North or East (upwards and rightwards in the map). There is also a one-way subway line that connects two intersections as shown by the dashed line. How many different paths can be used to get from the bottom left point to the top right point?
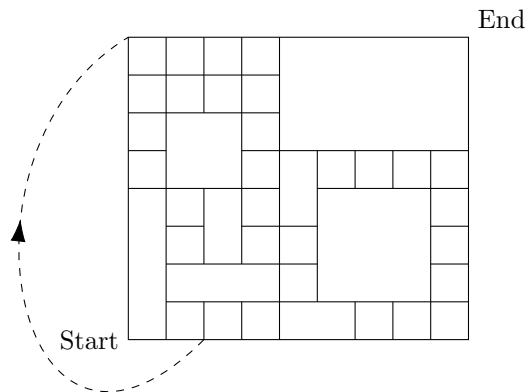


26. Two points are randomly chosen on a circle with radius 2. Find the probability that the distance from the center of the circle to the chord between the two points is at least $\sqrt{3}$. Express your answer as a **simplified** fraction.

27. Justin has a seven-digit positive integer consisting only of nonzero digits and the first digit is 7. Removing the first digit and adding it back at the end so that it becomes the last digit increases the number by 1691973. What is the original number?

28. Find the length of the segment indicated in the figure below. Express your answer as a decimal rounded to the nearest hundredth if necessary.

$?$    $5$

$7$
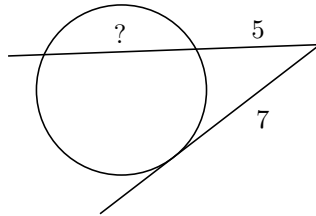
29. Find the number of digits of the smallest possible sum of two positive integers with a product of $10^{1000} - 1$.

30. If $a$ and $b$ are positive integers such that $\gcd(2a + 3b, a + 2b) = 12$ and $\operatorname{lcm}(ab^2, a^2b) = 1881792$, find $ab$.