

2020 Castro Valley Junior Math Tournament
Individual Solutions – 6th-8th Grades

1. **What number is 7 times the sum of 39 and 75?**

$$7(39 + 75) = 7 \cdot 114 = 798$$

2. **When the secret number is multiplied by 5 and this result is decreased by 27, the final result is 53. What is the secret number?**

The intermediate result must have been $53 + 27 = 80$, so that the secret number must have been $80 \div 5 = 16$.

3. **What digit is in the hundreds place of 4605.1797?**

The hundreds digit is the second digit of this number, for an answer of 6. Don't confuse it with the hundredTHs digit...

4. **What is the perimeter, in feet, of an equilateral triangle with sides measuring 3 ft?**

For an equilateral triangle, $P = 3s = 3 \cdot 3 = 9$.

5. **Evaluate: $-7 + 4(-6 - 7)$**

$$-7 + 4(-6 - 7) = -7 + 4(-13) = -7 - 52 = -59$$

6. **When one card is drawn from a standard 52-card deck, what is the probability that it is a red face card or the 9 of spades? Jacks, Queens, and Kings are considered face cards.**

We're hoping for the Jack, Queen, or King of Hearts, Jack, Queen, or King of Diamonds, or the 9 of Hearts, which is 7 cards out of a total of 52, for an answer of $\frac{7}{52}$.

7. **What is the 20th term of the Fibonacci Sequence, where each term is the sum of the two preceding terms, and beginning 1, 1, 2, 3, 5, 8, ...?**

It takes some time, but you need to make a list: 1, 1, 2, 3, 5, (FIVE) 8, 13, 21, 34, 55, (TEN) 89, 144, 233, 377, 610, (FIFTEEN) 987, 1597, 2584, 4181, 6765 (TWENTY).

8. **How many edges does a tetrahedron have?**

A tetrahedron has three edges around its triangular base, and three edges up to the top vertex, for a total of $3 + 3 = 6$.

9. **How many days are in 4 non-leap years?**

Because a non-leap year has 365 days, four of them will have $4 \cdot 365 = 1460$.

10. **What is the name for a triangle with a largest angle measuring 91 degrees?**

Because the angle is obtuse, the triangle is also called "obtuse".

11. **Express 0.9856 in scientific notation.**

$$.9856 = 9.856 \cdot .1 = 9.856 \cdot 10^{-1}$$

12. **What is the smallest number greater than 100 that leaves a remainder of 3 when divided by 7?**

7 goes into 70, 77, 84, 91, and 98, for an answer of $98 + 3 = 101$.

13. **What are the coordinates, in the form (x, y), of the y-intercept of the line $6x + 3y = 15$?**

The y-intercept occurs when $x = 0$, so $3y = 15$, giving $y = 5$, for an answer of (0, 5).

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14. **What is the units (ones) digit when 44703, 47940, 56391, and 68501 are added together?**

The units digit of the sum depends only on the units digits of the parts, for an answer of $3 + 0 + 1 + 1 = 5$.

15. **What is the sum of the 19 smallest counting numbers?**

There are $\frac{19}{2}$ “outer pairs” that each sum to $1 + 19 = 20 = 2 + 18 = \dots$, for an answer of $\frac{19}{2} \cdot 20 = 19 \cdot 10 = 190$.

16. **What is the length, in feet, of a leg of a right triangle with one angle measuring 45 degrees and a hypotenuse measuring 4 feet?**

A 45-45-90 triangle has sides in the ratio $x : x : x\sqrt{2}$, for an answer of $\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$.

17. **Fai is 5 times as old as Sebastian. 3 years ago, Sebastian was 2 years old. How old was Fai then?**

Sebastian is $3 + 2 = 5$ now, so Fai is $5 \cdot 5 = 25$ now, for an answer of $25 - 3 = 22$.

18. **A bag contains 9 yellow marbles and 9 green marbles. When two marbles are drawn, what is the probability that exactly 0 of them are green?**

There are $18c2 = \frac{18!}{2! \cdot 16!} = \frac{18 \cdot 17}{2} = 9 \cdot 17$ ways to choose two marbles out of 18 total marbles, and $9c2 = \frac{9!}{2! \cdot 7!} = \frac{9 \cdot 8}{2} = 9 \cdot 4$ ways to choose two yellow marbles, for a probability of $\frac{9 \cdot 4}{9 \cdot 17} = \frac{4}{17}$.

19. **Evaluate in terms of $i = \sqrt{-1}$: $(-1 + 8i)(1 - 3i)$**

FOIL gives $-1 + 3i + 8i - 24i^2 = -1 + 11i - 24(-1) = -1 + 11i + 24 = 23 + 11i$.

20. **What is the sum of the measures, in degrees, of the interior angles of a decagon?**

Drawing diagonals from a single vertex creates $10 - 2 = 8$ triangles with vertices at the vertices of the decagon, so the sum of the angles will be $8 \cdot 180 = 1,440$.

21. **The probability that it rains tomorrow is $\frac{1}{7}$, and the probability that I play video games tomorrow is $\frac{1}{8}$. If these two events are independent, what is the probability that it does NOT rain, but I DO play video games?**

$$\frac{6}{7} \cdot \frac{1}{8} = \frac{3}{7} \cdot \frac{1}{4} = \frac{3}{28}$$

22. **What are the coordinates, in the form (x, y) , of the leftmost x-intercept of the parabola with equation $y = x^2 - 18x - 33$?**

The quadratic formula gives zeros of $x = \frac{18 \pm \sqrt{324 + 132}}{2} = \frac{18 \pm 2\sqrt{81 + 33}}{2} = 9 \pm \sqrt{114}$, for an answer of $(9 - \sqrt{114}, 0)$.

23. **What is the range of the data set $\{5, 5, 0, 5\}$?**

The range is the difference between the largest and smallest element, giving $5 - 0 = 5$.

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24. **What value of c satisfies $2c + 41 = 7c - 24$?**

$65 = 5c$ yields $13 = c$.

25. **An equilateral triangle with an area of 5 m^2 is similar to another triangle, each of whose sides is 7 times as long as those of the original triangle. What is the area, in square meters, of the larger triangle?**

When planar (flat) shapes are similar, their lengths are r times one another's, and their areas are r^2 times one another's, for an answer of $5 \cdot 7^2 = 5 \cdot 49 = 245$.

26. **What is 25% of 132?**

$25\% = \frac{25}{100} = \frac{1}{4}$, for an answer of $\frac{132}{4} = \frac{66}{2} = 33$.

27. **What is the perimeter, in inches, of an equilateral triangle inscribed in a circle with a circumference of 20π in?**

In a circle, $C = \pi d = 2\pi r$, so $d = 20$ and $r = 10$. The inscribed triangle can be divided into six 30-60-90 triangles with hypotenuses that are radii of the circle and longer legs that are half a side of the equilateral triangle. If their hypotenuses are 10, their short legs are $10 \div 2 = 5$, and their long legs are $5\sqrt{3}$, for an answer of $6 \cdot 5\sqrt{3} = 30\sqrt{3}$.

28. **I order the \$20 value meal at my favorite restaurant, where sales tax is 10%. Because their service is so good, I always tip 20% of the bill with tax. How many dollars rounded to the nearest hundredth (cent) do I spend in total?**

The 10% tax is $20 \cdot \frac{10}{100} = 20 \cdot \frac{1}{10} = 2$, for a subtotal of $20 + 2 = 22$. The tip will then be $22 \cdot \frac{20}{100} = 22 \cdot \frac{2}{10} = \frac{44}{10} = 4.40$, for an answer of $22 + 4.40 = 26.40$.

29. **A cube of white plastic is painted blue on all sides, then cut into 729 congruent cubes. How many of these cubes are blue on exactly 2 faces?**

$729 = 9^3$, so the cube is cut into 9 slices in each dimension. The cubes that are blue on exactly two faces are on the edges of the original cube, but not the vertices. There are $9 - 2 = 7$ of these cubes on each edge of the larger cube, and there are 12 edges, for an answer of $7 \cdot 12 = 84$.

30. **Simplify by combining like terms: $9f + f^2 - 6 + 9f + 3f^2 + 8 - 7f^2$**

$f^2 + 3f^2 - 7f^2 + 9f + 9f - 6 + 8 = -3f^2 + 18f + 2$