2020 Castro Valley Junior Math Tournament
Geometry & Potpourri Solutions – 6th-8th Grades

1. **What is the perimeter, in feet, of a trapezoid with sides measuring 5 ft, 8 ft, 5 ft, and 10 ft?**

The perimeter of anything is the sum of its sides, for an answer of $5 + 8 + 5 + 10 = 28$.

2. **What is the smallest palindrome greater than 22969? A palindrome is a number that reads the same forwards or backwards, such as 121 or 3443.**

The logical first choice is 22922, but this is too small. The next larger palindrome would be 23032, which is the answer.

3. **What is the volume, in cubic centimeters, of a cube with edges measuring 5 cm?**

For anything that goes from flat to flat, $V = Bh$, and for a cube $V = s^2h = s^3 = 5^3 = 125$.

4. **What is the sum of the 18 smallest positive odd numbers?**

You can memorize that this is just $18^2 = 324$, but if you don't know that formula you can attack it as an arithmetic sequence. The smallest number will be 1, and the largest will be $1 + 17 \cdot 2 = 1 + 34 = 35$, so "outer pairs" will sum to $1 + 35 = 36 = 3 + 33 = \cdots$, there will be $18 \div 2 = 9$ such pairs, and the answer will be $9 \cdot 36 = 324$.

5. **How many diagonals can be drawn in a convex heptagon?**

Consider one of the seven vertices and the diagonals you can draw from it. You can draw a diagonal to every vertex other than yourself and your two neighbors, which is $7 - 3 = 4$ diagonals. Each of the 7 vertices would be the same, which you might think results in $7 \cdot 4 = 28$ diagonals. However, we've counted each diagonal from both of its ends, for a final answer of $28 \div 2 = 14$.

6. **Express the base-8 numeral 20_8 as a base-10 numeral.**

In base 8, the rightmost digit represents $8^0 = 1$s, the next digit represents $8^1 = 8$s, etc. Thus, $20_8 = 2 \cdot 8 + 0 \cdot 1 = 16 + 0 = 16$.

7. **What is the radius, in meters, of a circle with a circumference of 6π m?**

For a circle, $C = \pi d = 2\pi r$, so $6\pi = 2\pi r$, giving $6 = 2r$ and finally $3 = r$.

8. **What is the 7th term of the geometric (multiplying or dividing) sequence whose first three terms are 2, 6, 18?**

The common ratio is $6 \div 2 = 3 = 18 \div 6$, and the 7th term will be $7 - 1 = 6$ ratios from the first term, for an answer of $2 \cdot 3^6 = 2 \cdot 27^2 = 2 \cdot 729 = 1{,}458$.

9. **What is the length, in meters, of the hypotenuse of a right triangle with one angle measuring 30 degrees and a short leg measuring 2 meters?**

In a 30-60-90 triangle, the sides are in the ratio $x : x\sqrt{3} : 2$, for an answer of $2 \cdot 2 = 4$.

10. **Using the numbers 2, 4, and 1 exactly once each, and the operations of addition, subtraction, multiplication, and division (and parentheses) as much or as little as you like, create an expression that evaluates as close to 8 as possible. For example, if your digits were 4, 7, and 9, and your target were 8, your answer could be 4 x (9 - 7).**

For problems like this, looking for factors is often the best approach. In this case, the fact that 2 is a factor of 8 jumps out, as does the fact that 4 is a factor of 8, so multiplying is worth exploring. In this particular case, $4 \cdot 2 \cdot 1 = 8$ is one possible answer.

11. **What is the volume, in cubic feet, of a right circular cone with a base radius of 9 ft, and a height of 4 ft?**

For anything with a flat side going up to a point, $V = \frac{1}{3}Bh$. For a cone, this is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot 9^2 \cdot 4 = \pi \cdot 3 \cdot 9 \cdot 4 = \pi \cdot 27 \cdot 4 = 108\pi$.

12. **What is the most specific name that can be given to at least one quadrilateral with sides measuring 5 m, 7 m, 8 m, and 9 m in that order around its perimeter?**

It's clearly not a rhombus, parallelogram, or kite, because none of the sides are equal. Might it be able to be a trapezoid? If you consider the potential bases of 8 and 5, you can rotate the 9 and 7 sides so far that the 7 & 5 sides can align to make a 8-9-12 triangle on one extreme and so that the 9 & 5 sides can align to create a 7-8-14 triangle on the other extreme. Somewhere between these triangles the 5 side must have been parallel to the 8 base, so that a trapezoid is possible.

13. **What is the missing term of the sequence 1, 6, 16, 18, 31, 54, ___, ...?**

After trying lots of things and getting nowhere, you might consider that this sequence is actually two sequences that have been interspersed. 1, 16, 31, … is an arithmetic sequence with a common difference of $16 - 1 = 15 = 31 - 16$, and 6, 18, 54 is a geometric sequence with a common ratio of $18 \div 6 = 3 = 54 \div 18$, so that the answer is $31 + 15 = 46$.

14. **Two lines were initially perpendicular, but then I rotated one 77 degrees clockwise and the other 70 degrees counter-clockwise. What is the measure, in degrees, of the smaller angle between the lines now?**

Because the two lines are moving in opposite directions, their initial $90°$ angle is being changed by $70 + 77 = 147°$. $90 - 147 = -57°$, which just means the lines crossed one another and now have a $57°$ angle between them. The other angle would be $180 - 57 = 123°$, so the answer is 57.

15. **How many vertices does a regular icosahedron have?**

An icosahedron has 20 faces, each of which is an equilateral triangle, so you might think there would be $20 \cdot 3 = 60$ vertices. However, each vertex of an icosahedron is shared by five triangles, giving an answer of $60 \div 5 = 12$.

16. **What is the sum of the 14 smallest positive perfect squares?**

You could make a list and add them up, or there is the formula $\frac{n(n+1)(2n+1)}{6} = \frac{14 \cdot 15 \cdot 29}{6} = 7 \cdot 5 \cdot 29 = 35 \cdot 29 = 1{,}015$.

17. **What is the area, in square meters, of a right triangle with legs measuring 4 m and 12 m?**

A right triangle is half of a rectangle, so $A = \frac{1}{2}bh = \frac{1}{2} \cdot 4 \cdot 12 = 2 \cdot 12 = 24$.

18. **What is the largest number smaller than 1000 that leaves a remainder of 4 when divided by 6?**

900 is a multiple of 6, as are 990 and 996, for an answer of 994.

19. **What is the area, in square meters, of a circle inscribed in a square with an area of 16 m^2 ?**

If the square has an area of 16, its sides must be $\sqrt{16} = 4$. The diameter of the circle must also be 4, so its radius must be 2, for an answer of $A = \pi r^2 = \pi \cdot 2^2 = 4\pi$.

20. **How many three-digit counting numbers have a tens digit that is a multiple of 3 (0 doesn't count)?**

The hundreds digit can be anything from 1 to 9 (9 choices), the tens digit can be 3, 6, or 9 (3 choices), and the ones digit can be anything from 0 to 9 (10 choices), for an answer of $9 \cdot 3 \cdot 10 = 27 \cdot 10 = 270$.