2020 Castro Valley Junior Math Tournament
Geometry & Potpourri Solutions – 3rd-5th Grades

1. **What is the side length, in centimeters, of a square with a perimeter measuring 8 cm?**

$P = 4s$, so $8 = 4s$ and $s = 8 \div 4 = 2$.

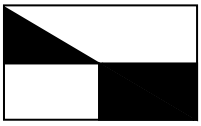
2. **What is the area, in square meters, of a rectangle with sides measuring 4 m and 7 m?**

$A = w \cdot l = 4 \cdot 7 = 28$

3. **What is the missing term of the geometric (multiplying or dividing) sequence 3, 21, 147, 1029, ___, ...?**

The common ratio is $21 \div 3 = 7 = 147 \div 21$, so the missing term will be $7 \cdot 1029 = 7{,}203$.

4. **What fraction of the figure to the right is UNshaded?**



The figure can be divided into four congruent rectangles, each of which could be divided into two triangles, which would be $2 \cdot 4 = 8$ triangles. Three of them are shaded, so five of them are not shaded, for an answer of $\frac{5}{8}$.

5. **What is the surface area, in square inches, of a cube with edges measuring 4 in?**

The surface of a cube is six squares, each with an area of $4 \cdot 4 = 16$, for an answer of $6 \cdot 16 = 96$.

6. **What is the measure in degrees, of an interior angle of an equilateral triangle?**

An equilateral triangle has three congruent sides, so three congruent angles. Since they must add up to 180°, each of them must be $180 \div 3 = 60$.

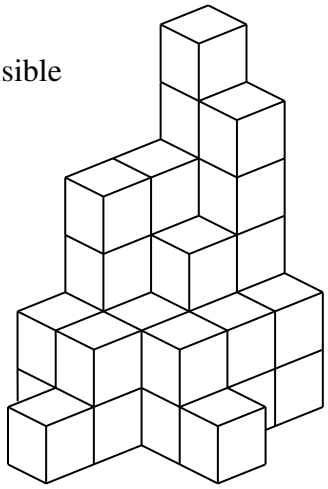
7. **How many counting numbers between 10 and 20 inclusive are prime?**

10, 12, 14, 16, 18, and 20 are divisible by 2, 11 is prime, 13 is prime, 15 is divisible by 3, 17 is prime, and 19 is prime, for an answer of 4.

8. **How many positive 2-digit counting numbers are palindromes? A palindrome is a number that reads the same forwards or backwards, such as 121 or 3443.**

The numbers in question are 11, 22, 33, …, 88, and 99, for a total of 9.

9. **What is the minimum number of blocks needed to build the stack shown to the right?**



The tallest stack is 6, then there's a 5, two 4s, a 3, six 2s, and two 1s, for a total of $6 + 5 + 2 \cdot 4 + 3 + 6 \cdot 2 + 2 \cdot 1 = 11 + 8 + 3 + 12 + 2 = 36$.

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10. **In a 3-by-3 magic square, the numbers 1-9 can be entered one per cell in such a way that the sum of each row, column, and diagonal is equal. If this is done in the magic square to the right, what is the product of c and d?**

	1	d
c		
2		

The numbers 1 to 9 add up to 45, so each row, column, and diagonal should add up to $45 \div 3 = 15$. The column with the 1 could be 5 & 9 or 6 & 8, and the row with the 2 could be 4 & 9 or 5 & 8 or 6 & 7, so the bottom middle number could be 9, 8, or 6, with the center number being 5, 6, or 8 and the lower right number being 4, 5, or 7. The upper left number could thus be 6, 4, or 0, so the third case isn't possible. d could thus be 8 or 10, so the second case isn't possible. Therefor $d = 8$ and $c = 7$, for an answer of 56.

11. **If the following set of six statements is logically consistent, which ones must be true? (e.g. A, B, & D)**

A. B is True
B. C is False
C. A & D are True
D. B & F are False
E. A & B are not both True
F. C & D are not both False

The best way to tackle these problems is usually to assume a statement is true (or false), and see what the results of that assumption are. Assuming A to be true, B would be true, in which case C would be false, in which case A & D could not both be true so D would be false (because A is already true), in which case B & F could not both be false, which matches the fact that B is true in this scenario. E talks about A & B, and is false because both A & B are true. F is similarly false, because both C & D are both false. This is a consistent set of results, so the answer is A & B. If the results hadn't been consistent, it would have meant that our original assumption of A being true was incorrect, so we would have to assume A was false and follow that train of logic instead.

12. **What is the measure, in degrees, of another interior angle of an isosceles triangle with one angle measuring 120 degrees?**

An isosceles triangle has two congruent sides, which means two congruent angles. The two angles cannot both be $120°$, because that would add up to $2 \cdot 120 = 240°$, which is more than the $180°$ that the angles of every triangle must add up to. So, the other two angles must be the equal angles, and they must add up to $180 - 120 = 60°$, so they are each $60 \div 2 = 30°$.

13. **What is the sum of the numbers below that are divisible by 5? 21, 170, 59, 15, 63, 48, 60**

Multiples of 5 have ones digits of 0 or 5, so the multiples of 5 are 170, 15, and 60, for a sum of 245.

14. **How many faces does a tetrahedron have?**

A tetrahedron is a triangular pyramid, and thus has four triangular faces.

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15. **How many diagonals can be drawn in a convex octagon?**

If you consider a single vertex of a polygon, it can have diagonals to all of the vertices except itself and its immediate neighbors. For this octagon, that means a single vertex can have $8 - 3 = 5$ diagonals. Because every one of the 8 vertices can have these 5 diagonals, one might think there are $8 \cdot 5 = 40$ diagonals, but this is not quite right. This number counts the diagonal from A to B from both the A-side and the B-side, and does this for every diagonal, so our answer is twice as big as it should be, for an answer of $40 \div 2 = 20$.

16. **Express the base-2 numeral 10_2 as a base-10 numeral.**

In base 2, the rightmost digit represents $2^0 = 1$s, the next digit represents $2^1 = 2$s, etc. In this case, $10_2 = 0 \cdot 1 + 1 \cdot 2 = 0 + 2 = 2_{10}$.

17. **A triangle has sides measuring 15 m, 42 m, and x m. What is the largest possible integer value of x?**

If the two known sides were pointing directly away from one another, they would span a distance of $15 + 42 = 57$, but they couldn't be the sides of a triangle, because they don't form an angle. If they have an angle very close to $180°$ between them, they'll make a third side just smaller than 57, which means the largest possible third side is 56.

18. **What is the units (ones) digit when 46987, 22828, 90644, and 37986 are added together?**

If we only care about the units digit of the answer, then we only care about the units digits of the numbers to be added. $7 + 8 + 4 + 6 = 15 + 10 = 25$, for an answer of 5.

19. **What is the sum of the 16 smallest counting numbers?**

The 16 numbers can be grouped into $16 \div 2 = 8$ "outer pairs", each with a sum of $16 + 1 = 17 = 15 + 2 = \cdots$, for a total of $8 \cdot 17 = 136$.

20. **What is the volume, in cubic feet, of a right rectangular prism (a box) with edges measuring 6 ft, 7 ft, and 8 ft?**

$V = l \cdot w \cdot h = 6 \cdot 7 \cdot 8 = 42 \cdot 8 = 336$