2020 Castro Valley Junior Math Tournament
Algebra & Probability Solutions – 6th-8th Grades

1. **What number is 47 more than the product of 8 and 62?**

$47 + 8 \cdot 62 = 47 + 496 = 543$

2. **When the special number is divided by 3 and this result is increased by 11, the final result is 34. What is the special number?**

The intermediate result must have been $34 - 11 = 23$, so that the special number must have been $23 \cdot 3 = 69$.

3. **A bag contains 5 red marbles, 6 orange marbles, 5 yellow marbles, 9 green marbles, and 5 blue marbles. When one marble is drawn at random, what is the probability that it is orange?**

There are a total of $5 + 6 + 5 + 9 + 5 = 30$ marbles, of which 6 are orange, for an answer of $\frac{6}{30} = \frac{1}{5}$.

4. **Evaluate as a mixed number:** $9\frac{2}{6} - 7\frac{1}{8}$

$9\frac{2}{6} - 7\frac{1}{8} = (9 - 7) + \left(\frac{2}{6} - \frac{1}{8}\right) = 2 + \left(\frac{1}{3} - \frac{1}{8}\right) = 2 + \left(\frac{8}{24} - \frac{3}{24}\right) = 2\frac{5}{24}$

5. **At a spelling bee with 5 participants, there is a huge first-place trophy, a modest second-place medal, and a tiny third-place certificate. In how many ways might these be awarded?**

There are 5 ways to award the trophy. No matter who gets it, there are 4 people who could get the medal, then 3 ways to award the certificate. The counting principle gives an answer of $5 \cdot 4 \cdot 3 = 20 \cdot 3 = 60$.

6. **Express 80530 in scientific notation.**

$80{,}530 = 8.053 \cdot 10{,}000 = 8.053 \cdot 10^4$

7. **When two cards are drawn from a standard 52-card deck, what is the probability that exactly 2 of them are 3s?**

There are $52c2 = \frac{52!}{2!\cdot 50!} = \frac{52\cdot 51}{2} = 26 \cdot 51$ ways to draw two cards from 52, and $4c2 = \frac{4!}{2!\cdot 2!} = \frac{4\cdot 3}{2} = 2 \cdot 3$ ways to draw two cards from 4 3s, for an answer of $\frac{2\cdot 3}{26\cdot 51} = \frac{1\cdot 1}{13\cdot 17} = \frac{1}{221}$.

8. **What is the discriminant of the quadratic $3x^2 - 3x - 7 = 0$?**

The discriminant is $b^2 - 4ac = (-3)^2 - 4 \cdot 3(-7) = 9 + 12 \cdot 7 = 9 + 84 = 93$.

9. **A bag contains 1 red marbles, 2 orange marbles, 9 yellow marbles, 8 green marbles, and 5 blue marbles. What is the smallest number of marbles I can grab without looking and be certain that I have selected at least two marbles of the same color?**

The worst thing I could get is one each of a bunch of different colors. There are five colors, so it's possible to draw five marbles and NOT get two of any color. At this point, however, any marble I draw will match one of the ones I already have, so 6 is the answer.

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10. **When the digits of a positive two-digit counting number are reversed, to create a new positive two-digit counting number, the new number is 9 more than the original number. What is the smallest possible value of the new number?**

11 reversed is still 11. 12 reversed is 21, which has a difference of 9, for an answer of 21.

11. **Arrange the letters below in order of ascending value (e.g. BCDA):**
A = $\frac{2}{5}$, B = 0.1, C = $\frac{1}{9}$, D = 0.61

$A = .4$, $B = .1$, $C \approx .11$, and $D = .61$, for an answer of BCAD.

12. **The point (4, -7) is rotated 540 degrees counter-clockwise about the point (0, -5). What are its new coordinates, in the form (x, y)?**

(4, -7) is 4 units to the right of (0,-5) and 2 units below it. $540 = 360 + 180$, so it is equivalent to $180°$. Rotating $180°$ puts the point directly across from where it started, so it should be 4 units to the left and 2 units above (0, -5), for an answer of (-4, -3).

13. **When two fair coins are flipped, what is the probability that they show exactly 1 tail?**

When two coins are flipped, you can get HH, HT, TH, or TT, which is four possibilities. Of these, two have 1 tail, for an answer of $\frac{2}{4} = \frac{1}{2}$.

14. **If it is currently 4:27 PM, what time was it 727 minutes ago? Include AM or PM.**

Every 60 minutes is an hour, so 727 minutes is $727 \div 60 = 12\frac{7}{60}$ hours, which is 12 hours and 7 minutes. 12 hours ago is 4:27 AM, and 7 minutes earlier is 4:20 AM.

15. **A bag contains 3 red marbles, 7 green marbles, and 2 yellow marbles. A trusted friend draws a single marble, looks at it, and tells you it is not red. What is the probability that the marble is green?**

If it's not red, then there are only $7 + 2 = 9$ marbles it could be, 7 of which are green, for a probability of $\frac{7}{9}$.

16. **If 144 chickens can lay 60 eggs in 8 days, how many chickens would it take to produce 360 eggs in 16 days?**

360 eggs is $360 \div 60 = 6$ times 60 eggs, so it would require 6 times as many chickens in the same amount of time, which would be $6 \times 144 = 864$ chickens. 16 days is $16 \div 8 = 2$ times 8 days, so it would require $\frac{1}{2}$ as many chickens, for an answer of $864 \div 2 = 432$.

17. **When two standard six-sided dice are rolled, what is the probability that exactly 0 of them show(s) a number greater than 4?**

The probability that the red one is less than or equal to 4 is $\frac{4}{6} = \frac{2}{3}$, and the same is true for the blue one, so the probability that both of them are is $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$.

18. **Arrange the letters below in order of ascending value (e.g. BCDA):**
A = 2 x 2, B = $1 \div 3$, C = 4 + 7, D = 2 – 5

$A = 4$, $B = \frac{1}{3}$, $C = 11$, and $D = -3$, for an answer of DBAC.

19. **Express the range of $m(k) = 5 - \sqrt{7n + 2}$ in interval notation, give that both the domain and range are subsets of the real numbers.**

The range is the possible y-values (m-values in this case). The square root function can output anything from 0 to infinity, so when it is subtracted from 5 the result will be anything from 5 down to negative infinity, which is written $(-\infty, 5]$.

20. **What is the solution, in the form (w, v, u), of the system of equations w + v = 4, w + u = 0, and u + v = 6 ?**

If you add all three equations, you get $2w + 2v + 2u = 10$, then $w + v + u = 5$. Subtracting each original equation from this gives $u = 1$, $v = 5$, and $w = -1$, for an answer of $(-1, 5, 1)$.